LOGIC COLLOQUIUM 2018

July 23–28
Udine, Italy

lc18.uniud.it

Program and Abstracts

The event is made possible thanks to the financial support of...

Dip. Scienze Matematiche Informatiche e Fisiche–Università di Udine

Association for Symbolic Logic

Istituto Nazionale di Alta Matematica-GNSAGA

National Science Foundation

Società Italiana di Logica e Filosofia delle Scienze

Associazione Italiana di Logica e sue Applicazioni

Italian Chapter of the EATCS

UNIVERSITÀ DEGLI STUDI DI UDINE
hic sunt futura

Sponsor
ENERGIA & SERVIZI AMGA
HERAcomm

Programme and Abstracts

Logic Colloquium 2018

Udine, Italy
July 23-28, 2018

Contents

Preface

The LOGIC COLLOQUIUM 2018 is the annual European summer meeting of the Association of Symbolic Logic (ASL), that will be held during July 23–28, 2018 at the University of Udine, Italy.

Logic is an ancient discipline that has undergone striking modern developments through the introduction of formal methods, stimulated largely by foundational problems in mathematics. Recent research in other areas such as computer science, linguistics, and cognitive science has also been inspired by logic, and the LOGIC COLLOQUIUM reflects such expanding interests.

The scientific programme of LC 2018 consists of:

- 2 tutorials, given by
  - Ulrike Sattler (University of Manchester)
  - Katrin Tent (WWU Münster)
- the Gödel lecture, given by
  - Rod Downey (Victoria University of Wellington)
- 11 plenary lectures, given by
  - Marianna Antonutti Marfori (Ludwig-Maximilians-Universität München)
  - Albert Atserias (Universitat Politècnica de Catalunya)
  - Vasco Brattka (Universität der Bundeswehr München)
  - Agata Ciabattoni (TU Wien)
  - Paola D'Aquino (Università degli Studi della Campania)
  - Paulo Oliva (Queen Mary University of London)
  - Ludovic Patey (Institut Camille Jordan, Lyon)
  - Anush Tserunyan (University of Illinois at Urbana-Champaign)
  - Spencer Unger (Tel Aviv University)
  - Matteo Viale (Università degli Studi di Torino)
  - Dag Westerståhl (Stockholm University)
- 6 special sessions with a total of 24 talks, as follows:
  - Special session on computability theory, with speakers:
    - Johanna Franklin (Hofstra University)
    - Takayuki Kihara (Nagoya University)
    - Keng Meng Ng (Nanyang Technological University)
    - Linda Brown Westrick (University of Connecticut)
  - Special session on descriptive set theory and dynamical systems, with speakers:
    - Clinton Conley (Carnegie Mellon University)
    - Julien Melleray (Université Lyon I)
    - Todor Tsankov (Université Paris Diderot)

- Robin Tucker-Drob (Texas A&M University)
- Special session on model theory, with speakers:
  - Gabriel Conant (University of Notre Dame)
  - Adrien Deloro (Université Pierre et Marie Curie)
  - Nadja Hempel (UCLA)
  - Nick Ramsey (UC Berkeley)
- Special session on philosophy of logic and mathematics, with speakers:
  - Brice Halimi (Université Paris Nanterre)
  - Simon Hewitt (University of Leeds)
  - Gil Sagi (University of Haifa)
  - Nicole Wyatt (University of Calgary)
- Special session on proof theory, with speakers:
  - Ryota Akiyoshi (Waseda University)
  - Martin Escardó (University of Birmingham)
  - Alessandra Palmigiano (TU Delft)
  - Chuangjie Xu (Ludwig-Maximilians-Universität München)
- Special session on temporal and multivalued logics, with speakers:
  - Agi Kurucz (King's College London)
  - Daniele Mundici (Università degli Studi di Firenze)
  - Paritosh K. Pandya (Tata Institute of Fundamental Research; IIT Mumbai)
  - Amanda Vidal (Czech Academy of Sciences)
- 97 contributed talks.

The event is made possible thanks to the financial support of:

- Università degli Studi di Udine
- Dipartimento di Scienze Matematiche, Informatiche e Fisiche
- Association for Symbolic Logic
- Istituto Nazionale di Alta Matematica - GNSAGA
- National Science Foundation
- Associazione Italiana di Logica e sue Applicazioni
- Società Italiana di Logica e Filosofia delle Scienze
- European Association for Theoretical Computer Science, Italian Chapter

and our sponsor

LC 2018 committees

Program committee

- Dugald Macpherson, Chair (University of Leeds)
- Stéphane Demri (CNRS)
- Alexander S. Kechris (California Institute of Technology)
- Chris Laskowski (University of Maryland)
- Alberto Marcone (Università degli Studi di Udine)
- Antonio Montalban (UC Berkeley)
- Pavel Pudlák (Czech Academy of Sciences)
- Gila Sher (UC San Diego)
- Dima Sinapova (University of Illinois at Chicago)

Local organizing committee

- Giovanna D'Agostino, Co-chair (Università degli Studi di Udine)
- Angelo Montanari, Co-chair (Università degli Studi di Udine)
- Vincenzo Dimonte (Università degli Studi di Udine)
- Guido Gherardi (Università di Bologna Alma Mater)
- Alberto Marcone (Università degli Studi di Udine)
- Franco Parlamento (Università degli Studi di Udine)
- Carla Piazza (Università degli Studi di Udine)
- Dario Della Monica (Istituto Nazionale di Alta Matematica "F. Severi")
- Marta Fiori Carones (Università degli Studi di Udine)
- Emanuele Frittaion (University of Lisbon)
- Nicola Gigante (Università degli Studi di Udine)
- Alberto Molinari (Università degli Studi di Udine)
- Manlio Valenti (Università degli Studi di Udine)

	Monday	Tuesday
9.00-10.00	Vasco Brattka	Anush Tserunyan
10.30–11.30	Ulrike Sattler	Ulrike Sattler
11.30-12.30	Paulo Oliva	Paola D'Aquino

	Wednesday	Thursday
9.00-10.00	Rod Downey	Matteo Viale
10.30–11.30	Ulrike Sattler	Katrin Tent
11.30-12.30	Katrin Tent	Albert Atserias

	Friday	Saturday
9.00-10.00	Agata Ciabattoni	Ludovic Patey
10.30–11.30	Katrin Tent	Marianna Antonutti-Marfori
11.30-12.30	Dag Westerståhl	Spencer Unger

Tutorials

Uli Sattler

Description logics, ontologies, and automated reasoning: an introduction

Description logics (DL) [1, 2] form the logical basis of state-of-the-art ontology languages, in particular the Semantic Web Ontology language OWL [3]. They have been first developed as the formalisation of semantic networks and frames, and are "coincidental" cousins of modal logic and the guarded fragment, and hence decidable fragments of first order logic.

In the last three decades, we have seen a wide range of contributions and applications, due to mutually beneficial interactions between the following areas of activity:

- variants, extensions and combinations of description logics being investigated with respect to their decidability, computational complexity, model theory, and other relevant properties;
- automated reasoners being developed, constantly optimised to cater for ever more demanding application scenarios, extended to cater for a wide range of reasoning tasks, and supported by other tools;
- tools such as editors, integrated development environments, and programmatic APIs being developed and constantly improved that integrate well with reasoners and support domain experts in modelling;
- applications–in particular from bio-health applications but also from other knowledge-heavy domains–that benefit from the "semantic lense" that description logic theories provide and the reasoning services offered via DL reasoners, and that require novel, non-standard reasoning services such as module extraction or entailment explanations.

This development was helped by the standardisation of the syntax and extensions to this syntax to annotate/comment logical theories and axioms with relevant book-keeping information.

In this tutorial, I will give an introduction to description logics, their relationship to modal and first order logics, and the four areas highlighted above. This introduction is aimed at anybody with a general background in logic and an interest in learning more about the field of description logics, knowledge representation, and ontology engineering.

References

[1] F. Baader, D. Calvanese, D. McGuinness, D. Nardi, and P. F. Patel-Schneider, ***The Description Logic Handbook: Theory, Implementation, and Applications.***, Cambridge University Press, 2003.

[2] F. Baader, I. Horrocks, C. Lutz, and U. Sattler, ***Introduction to Description Logic***, Cambridge University Press, 2017.

[3] B. Cuenca Grau, I. Horrocks, B. Motik, B. Parsia, P. F Patel-Schneider, and U. Sattler, *OWL 2: The next step for OWL*, ***Journal of Web Semantics***, vol. 6 (2008), no. 4, pp. 309–322.

School of Computer Science, University of Manchester, UK
uli.sattler@manchester.ac.uk

Katrin Tent

Model theoretic ampleness

The notion of ampleness originates in algebraic geometry and characterizes an embeddability property into projective spaces.

This notion turned out to be crucial in the characterization of Zariski geometrie due to Hrushovski and Zilber.

Pillay subsequently introduced a model theoretic version of ampleness. His definition can be seen as an attempt to characterize projective spaces just using model theoretic independence. It remained an open question whether ampleness might be sufficient to characterize a strongly minimal structure as being an algebraic curve.

In this series of lectures, I will give an introduction to model theoretic independence and explain the definition of ampleness. I will then go on to explain a number of different examples of ample structures, starting with projective spaces and other kinds of buildings, sketching ampleness in the free group. Finally I will explain recent constructions of ample strongly minimal structures not arising as an algebraic curve.

This work is partially supported by SFB 878.

Mathematisches Institut, Universität Münster, Einsteinstrasse 62, 48149 Münster, Germany
tent@wwu.de

Gödel Lecture and Plenary Talks

Rod Downey

Algorithmic Randomness

In spite of the fact that elementary probability theory tells us that all sequences of n tosses of a fair coin are equally likely, our intuition tells us that some sequences are more random than others. Is there a reasonable mathematical theory of randomness of individual objects rather than one of expected behavior of distributions? In this talk I will discuss work in the area of mathematics devoted to interpreting randomness through computation. I will begin with Borel, von Mises and Turing and finish discussing some things we have learned in recent years. The lecture should be accessible to graduate students.

School of Mathematics, Statistics & Operations Research Victoria University of Wellington P.O. Box 600, Wellington, New Zealand
rod.downey@vuw.ac.nz

Vasco Brattka

On the computational content of theorems

To analyze the computational content of theorems is a research topic at least since Turing's seminal work on computable numbers in which he started the investigation of computable versions of theorems in analysis. In the sequel this topic was taken up by many other researchers such as Specker, Lacombe, Shore and Nerode, Pour-El and Richards [2], and Weihrauch [1]. A related but formally different approach has been started by Friedman and Simpson [3], who have characterized axioms that are sufficient and often necessary to prove certain theorems in second-order arithmetic. This approach is best known under the name reverse mathematics. In recent years the interaction between these two research trends has been intensified and overlaps in what is called Weihrauch complexity. Weihrauch complexity is a computability theoretic approach to the classification of the computational content of theorems that yields results that can be seen as a uniform and resource sensitive version of reverse mathematics. The benefit of this theory is that it yields fine grained computational results that answer typical questions from the computable analysis perspective, while being compatible with reverse mathematics. Sometimes results can be imported from reverse mathematics and computable analysis, but often completely new methods and techniques are required. We will present a survey on this approach that is based on a recent survey article [1] on this topic.

References

[1] V. Brattka, G. Gherardi, and A. Pauly, ***Weihrauch complexity in computable analysis***, arXiv 1707.03202, 2017.

[2] M. B. Pour-El and J. I. Richards, ***Computability in Analysis and Physics***, Springer, Berlin, 1989.

[3] S. G. Simpson, ***Subsystems of Second Order Arithmetic***, Cambridge University Press, Poughkeepsie, 2. ed., 2009.

[4] K. Weihrauch, ***Computable Analysis***, Springer, Berlin, 2000.

Department of Computer Science, Universität der Bundeswehr München, 85577 Neubiberg, Germany and Department of Mathematics and Applied Mathematics, University of Cape Town, Rondebosch 7700, South Africa

E-mail address

Paulo Oliva

Relational proof interpretations

Functional interpretations come in two flavours: one either interprets formulas as *sets* of realisers, or as *relations* between potential realisers and counter-realisers. In the first group we have the various realizability interpretations, such as Kleene's original numerical realizability, or Kreisel's modified realizability. In the second class we have Dialectica-like interpretations, such as Gödel's functional interpretation or its Diller-Nahm variant. In this talk we review the observation of [2] that a relational version of modified realizability also exists, and that its set-based definition can be derived from the relational one. We stress the advantages of this (more general) relational approach: it allows for a unification of realizability and Dialectica interpretations, enabling a modified realizability of linear logic [3], and a hybrid interpretation that combines various interpretations [4]. Surprisingly, this even includes truth-variants of these interpretations [1], and explains these in terms of the linear logic exponentials. We conclude by discussing recent work on a similar (relational) generalisation and unification for type-free (partial) interpretations such as Kleene's original realizability.

References

[1] Jaime Gaspar and Paulo Oliva, *Proof interpretations with truth*, ***Mathematical Logic Quarterly***, vol. 56 (2010), no. 6, pp. 591–610.

[2] Paulo Oliva, *Unifying functional interpretations*, ***Notre Dame Journal of Formal Logic***, vol. 47 (2006), no. 2, pp. 263–290.

[3] ——— *Modified realizability interpretation of classical linear logic*, ***Proc. of the Twenty Second Annual IEEE Symposium on Logic in Computer Science LICS'07*** (Wroclaw, Poland), IEEE Press, 2007, pp. 431–442.

[4] ——— *Hybrid functional interpretations of linear and intuitionistic logic*, ***Journal of Logic and Computation***, vol. 22 (2012), no. 2, pp. 305–328.

School of Electronic Engineering and Computer Science, London, UK
`p.oliva@qmul.ac.uk`

Anush Tserunyan

Ergodic theorems and descriptive combinatorics

"Gee Professor Kechris, descriptive set theory sure is powerful, and beautiful too!" was my friend's suggestion on how to phrase my email to my future PhD advisor asking for a reading course. I still believe this, of course, and I'll try to convince you as well by exhibiting an example of how modern descriptive set theoretic thinking combined with combinatorial and measure theoretic arguments yields a pointwise ergodic theorem for quasi-probability-measure-preserving locally countable graphs. This can be viewed as a general analogue of pointwise ergodic theorems for group actions, as a group action naturally induces a graph, its Schreier graph. The theorem states that ergodicity (indecomposability) of a graph amounts to locally approximating global averages of L^1-functions via increasing subgraphs with finite connected components.

Department of Mathematics, University of Illinois at Urbana-Champaign, 1409 W Green St Urbana, IL, 61801, USA
anush@illinois.edu

Paola D'Aquino

Complex Exponential Field

In his paper [2] Zilber identifies a new class of exponential fields (pseudo-exponential fields), proves a categoricity result in every uncountable cardinality, and puts forward the dramatic conjecture that the classical complex exponential field is the unique model of power continuum. The huge importance of this conjecture for the classical case is that Zilber has, unconditionally, established, for the pseudo-exponential fields, geometrically natural criteria for solvability of systems of exponential equations, whereas in the classical case only a very few such criteria have been established by using hard complex analysis, for example Nevanlinna Theory. In the last 15 years much attention has been given to extend classical results for the complex exponential field to the pseudo-exponential fields, and vice versa much effort has been put in proving for $\mathbb{C}$ properties of solutions of exponential polynomials which follow from the axioms of Zilber. The analytic methods have been substituted by algebraic and diophantine-geometrical arguments. I will review some of the first results on this and I will present some more recent achievements obtained in collaboration with A. Fornasiero and G. Terzo

References

[1] P. D'Aquino, A. Fornasiero and G. Terzo *Generic solutions of equations with iterated exponentials*, ***Transactions of American Mathematical Society***, vol. 370, (2018), no. 2, pp. 1393–1407.

[2] B. Zilber *Pseudo-exponentiation on algebraically closed fields of characteristic zero*, ***Annals of Pure and Applied Logic***, vol. 132, (2004), no. 1, pp. 67–95.

Dipartimento di Matematica e Fisica, Università della Campania "L. Vanvitelli", viale Lincoln 5, Caserta 81100, Italy
`paola.daquino@unicampania.it`

Matteo Viale

Forcing as a tool to prove theorems

Forcing is a fundamental working tool of set theorists, and is the standard method to obtain independence results. The aim of this talk is to outline that forcing can also be used to find the correct solution for certain types of mathematical problems. Specifically we will present two examples of metamatemathical arguments (based on generic absoluteness results and forcing) which can be used to answer two rather concrete questions, one related to Schanuel's conjecture on the transcendence properties of the complex exponential, and the other related to the classification problem for countable abelian groups [1, 3]. We will also give a general picture of the generic absoluteness results mainly by Woodin (for second-order arithmetic) and myself (for fragments of third-order arithmetic in combination with forcing axioms) [2] and their connections with large cardinals, forcing axioms, and with other non-constructive principles such as the axiom of choice and the Baire category theorem.

References

[1] F. Calderoni and S. Thomas *The bi-embeddability relation for countable abelian groups.* ***Transactions of the AMS***, DOI: https://doi.org/10.1090/tran/7513

[2] M. Viale, *Category forcings,* MM^{+++}*, and generic absoluteness for the theory of strong forcing axioms.*, ***Journal of the AMS***, vol. 29 (2016), no. 3, pp. 675–728.

[3] ———, *Forcing the truth of a weak form of Schanuel's conjecture*, ***Confluentes Mathematicae***, vol. 8 (2016), no. 2, pp. 59–83.

Department of Mathematics "Giuseppe Peano", University of Torino, Via Carlo Alberto 10, Torino, Italy
`matteo.viale@unito.it`

Albert Atserias

What can not be solved by the ellipsoid method?

The ellipsoid method, developed in the 1960's for non-linear programming, and rigorously analyzed by Khachiyan in the 1970's for linear programming, is a powerful algorithm for solving linear optimization problems over convex sets that are given by a separation oracle. One of the important features of the method is that it gives a polynomial-time algorithm for solving not only explicitly given linear programs, but also certain implicitly given exponentially big linear or semidefinite programs that arise in combinatorial contexts. The massive flexibility of this method turns the following question into a challenge: what are the limits of the ellipsoid method? In other words, what are the combinatorial problems that the ellipsoid method can provably *not* solve in polynomial time? In this talk we will show how the methods of mathematical logic, concretely, the methods of descriptive complexity and finite model theory, provide good answers to some of these questions.

Computer Science Department, Universitat Politècnica de Catalunya, Barcelona, Spain
atserias@cs.upc.edu

Agata Ciabattoni

Analytic calculi for substructural logics: theory and applications

ubstructural logics are axiomatic extensions of full Lambek calculus. They encompass, among many others, classical, intuitionistic, intermediate, fuzzy, and relevant logics. In my talk I will outline some results towards a uniform and systematic introduction of cut-free sequent and hypersequent calculi for substructural logics.

The calculi are defined by integrating proof theoretic and algebraic techniques, starting from Hilbert systems [4, 5, 6].

The hypersequent calculi are used to provide a concurrent computational interpretation for many intermediate logics, classical logic included. This solves Avron's conjecture [3]. We use indeed the Curry–Howard correspondence to obtain new typed concurrent λ-calculi, each of which features a specific communication mechanism and implements forms of code mobility [1, 2].

References

[1] F. Aschieri, A. Ciabattoni and F. A. Genco, *Gödel logic: From natural deduction to parallel computation*, ***Proceedings of Logic in Computer Science*** (LICS 2017), pp. 1 – 12, 2017.

[2] ——— *Classical Proofs and Parallel Programs*, ***Submitted*** 2018.

[3] A. Avron, *Hypersequents, logical consequence and intermediate logics for concurrency*, ***Annals of Mathematics and Artificial Intelligence***, vol. 4, pp. 225-248, 1991.

[4] A. Ciabattoni, N. Galatos and K. Terui, *Algebraic proof theory: hypersequents and hypercompletions*, ***Annals of Pure and Applied Logic***, vol. 168, no. 3, pp. 693-737, 2017.

[5] ——— *Algebraic proof theory for substructural logics: cut-elimination and completions*, ***Annals of Pure and Applied Logic***, vol. 163, no. 3, pp. 266-290, 2012.

[6] ——— *From axioms to analytic rules in nonclassical logics*, ***Proceedings of Logic in Computer Science*** (LICS 2008), pp. 229 – 240, 2008.

Department of Logic and Computation, Theory and Logic group, Vienna University of Technology, Favoritenstrasse 9-11, Austria
agata@logic.at

Dag Westerståhl

Logical constants and logical consequence

Logical consequence is a (*the*?) central notion in logic, and consequence relations in general and their properties is a familiar topic in the discipline. The dual notion of a logical constant has received less attention. Yet consequence depends on constants: as Bolzano and (100 years later) Tarski made precise, every choice of a set X of constants yields a corresponding consequence relation $\Rightarrow_X$, defined in terms of truth preservation under replacement/reinterpretation of symbols *outside* X. How to choose X? Both Bolzano and Tarski emphasized the importance of this question, and admitted they had no answer (at the time; Tarski later suggested an answer in terms of permutation invariance). However, our intuitions about 'what follows from what' are stronger than those about what is constant, and it makes sense to try to see how constants depend on consequence, by 'extracting' the constants from a given consequence relation. A natural way to make this precise yields a Galois connection establishing the duality between sets of constants and Bolzano/Tarskian consequence relations.

Logical consequence relations are also defined syntactically, via rules. If such a relation coincides with the Bolzano/Tarskian consequence relation given by the standard logical symbols with their standard interpretation, we have (soundness and) *completeness*: syntax matches the semantics. But the other side of the coin, that semantics matches syntax in the sense that the meaning of the logical symbols is (at least to a large extent) determined by the relation of logical consequence, seems fundamental too. Such *categoricity* results were the main focus of Carnap's 1943 book *The Formalization of Logic*, but the topic has been rather neglected since.

I will report on ongoing work together with Denis Bonnay on these issues, first presenting the abstract relations between constants and consequence, and then some concrete categoricity results (work begun in [1, 2]). While the picture for first-order logic and its extensions is fairly clear, new conceptual issues, and some open questions, arise in the case of other logics such as modal logic.

References

[1] Denis Bonnay and Dag Westerståhl, *Consequence mining. Constants versus consequence relations*, ***Journal of Philosophical Logic***, vol. 41 (2012), no. 4, pp. 671–709.

[2] ——— *Compositionality solves Carnap's problem*, ***Erkenntnis***, vol. 81 (2016), no. 4, pp. 721–739.

Department of Philosophy, Stockholm University, 106 91 Stockholm, Sweden
dag.westerstahl@philosophy.su.se

Ludovic Patey

Ramsey's theorem from a computable perspective

Ramsey's theorem for n-tuples an k colors (RT^n_k) asserts that given a k-coloring of $[\mathbb{N}]^n$, there exists an infinite set H such that $[H]^n$ is monochromatic. This theorem is not computably true, in that there is a computable such coloring with no infinite computable monochromatic set. Ramsey's theorem can be seen as a mathematical problem, in terms of *instances* and *solutions*. An instance is a k-coloring f of $[\mathbb{N}]^n$, and a solution to f is an infinite set H such that $[H]^n$ is monochromatic. A natural question to ask is "how hard is it to compute a solution given an instance of Ramsey's theorem?" The study of the computational strength of Ramsey's theorem is a long journey, started with Carl Jockusch, with high feats, such as Seetapun's theorem and Liu's theorem. This talks aims to be a gentle introduction to the modern analysis of Ramsey's theorem from the viewpoint of proof-theory, computability theory and Weihrauch degrees. These approaches happened to be a very fruitful line of research, leading to the development of new techniques in various areas. Some important questions remain however open, and we shall stress the remaining challenges.

Institut Camille Jordan - CNRS
Ludovic.Patey@computability.fr

Marianna Antonutti-Marfori

On the Significance of Mathematical Hierarchies

The study of hierarchies in mathematics has been a very active field of research since the 1940s. Hierarchies are usually taken to classify objects (or collections thereof) according to a measure of complexity along some dimension, such that the level at which a given object appears in the hierarchy measures the degree of difficulty of constructing the object in question, or equivalently, the difficulty of verifying its existence. Theories considered mathematically 'natural' often occupy special places in these hierarchies, and can sometimes be put into interesting correspondences with foundational or philosophical approaches to mathematics, according to the strength of their existential assumptions.

In this talk, I will propose a different, complementary view of the lessons we can draw from the study of mathematical hierarchies. According to this view, hierarchies measure the relative distance from the axiomatic assumptions that we make on the basis of our pre-theoretical understanding of a certain domain of mathematical objects, by means of countable or uncountable iterations of inference patterns that we recognise as correct. In outlining this view, I will consider how the study of hierarchies developed from the new formal analyses of concepts such as computation, consistency, interpretation, and model that emerged in the early 20th century, and I will suggest that a deeper understanding of the historical development of mathematical hierarchies can help illuminate their significance.

Ludwig-Maximilians Universität München
marianna.antonutti@gmail.com

Spencer Unger

Stationary reflection

joint work with Yair Hayut

Stationary reflection is an important notion in the study of compactness principles in set theory. The existence of a nonreflecting stationary set is enough to construct many objects which witness the noncompactness of various properties, examples include noncompactness of the chromatic number of graphs and the extent of freeness of abelian groups. In the other direction, stationary reflection (the assertion that every stationary set reflects) is consistent relative to the existence of large cardinals.

At the successor of a singular cardinal, the previous known upperbound was infinitely many supercompact cardinals. In particular, Magidor showed that stationary reflection at $\aleph_{\omega+1}$ is consistent from this hypothesis. We improve this upperbound by showing that stationary reflection at $\aleph_{\omega+1}$ is consistent relative to the existence of a cardinal κ which is κ^+-Π^1_1-subcompact. Under GCH this large cardinal assumption follows from κ^+-supercompactness.

School of Mathematical Sciences, Tel Aviv University, Tel Aviv 69978, Israel
spencerunger@mail.tau.ac.il

Special Sessions

Monday

	Computability Theory	**Model Theory**	**Philosophy of Logic and Mathematics**
14:30-15:15	Johanna Franklin	Gabriel Conant	Nicole Wyatt
15:15-16:00	Takayuki Kihara	Adrien Deloro	Simon Hewitt

Tuesday

	Descriptive Set Theory	**Proof Theory and Constructivism**	**Temporal and Multivalued Logics**
14:30-15:15	Todor Tsankov	Alessandra Palmigiano	Agi Kurucz
15:15-16:00	Robin Tucker-Drob	Chuangjie Xu	Daniele Mundici

Thursday

	Computability Theory	**Model Theory**	**Philosophy of Logic and Mathematics**
14:00-14:45	Linda Brown Westrick	Nadja Hempel	Gil Sagi
14:45-15:30	Kang Meng Ng	Nick Ramsey	Brice Halimi

Friday

	Descriptive Set Theory	**Proof Theory and Constructivism**	**Temporal and Multivalued Logics**
14:00-14:45	Julien Melleray	Martin Escardó	Paritosh K. Pandya
14:45-15:30	Clinton Conley	Ryota Akiyoshi	Amanda Vidal

Johanna N.Y. Franklin

Lowness and computable metric spaces

joint work with Timothy H. McNicholl

In the past five years, lowness, a familiar notion in other areas of computability theory, has been introduced into computable structure theory; specific types of structures studied include equivalence structures and various types of linear orders. Recently, we have extended this work to computable metric spaces. Here, we present some results on lowness for computably presented metric spaces and then, more specifically, Banach spaces.

Department of Mathematics, Hofstra University, Room 306, Roosevelt Hall, Hempstead, NY 11549-0114, USA
johanna.n.franklin@hofstra.edu

Takayuki Kihara

On the structure of the Wadge degrees of BQO-valued Borel functions

joint work with Antonio Montalbán

In this talk, we give a full description of the Wadge degrees of Borel functions from ω^ω to a better-quasi-ordering $\mathcal{Q}$. More precisely, for any countable ordinal ξ, we show that the Wadge degrees of $\underset{\sim}{\Delta}^0_{1+\xi}$-measurable functions $\omega^\omega \to \mathcal{Q}$ can be represented by countable joins of the ξ-th transfinite nests of $\mathcal{Q}$-labeled well-founded trees [2]. This result generalizes and unifies former works by Wadge, Duparc, Selivanov, and others. This also has a consequence in computability theory, since it is shown that there is a natural isomorphism between the structure of the Wadge degrees of $\mathcal{Q}$-valued functions and that of the "natural" many-one degrees of $\mathcal{Q}$-valued problems [1].

Our main theorem is completely described in the descriptive-set-theoretic language; nevertheless our proof requires various computability-theoretic tools such as Marcone-Montalbán's *Turing jump operator via true stages*, and (the uniform version of) the *Friedberg jump inversion theorem*. Thus our work provides a new application of computability theory to descriptive set theory.

References

[1] Takayuki Kihara, and Antonio Montalbán, *The uniform Martin's conjecture for many-one degrees*, ***Transactions of the American Mathematical Society***, to appear.

[2] Takayuki Kihara, and Antonio Montalbán, *On the structure of the Wadge degrees of BQO-valued Borel functions*, submitted.

Graduate School of Informatics, Nagoya University, 1 Furo-cho, Chikusa-ku, Nagoya, 464-0814 Japan
kihara@i.nagoya-u.ac.jp

LINDA BROWN WESTRICK

Topology and parallelized Weihrauch reducibility

joint work with Takayuki Kihara

The parallelized Weihrauch reducibility on functions $f, g : 2^\omega \to \mathbb{R}$ is defined as follows: $f \leq_W \hat{g}$ if and only if f is Weihrauch reducible to ω-many copies of g. We give some alternate characterizations of this and related notions, relating them to the choice of topology on 2^ω and on the space of functions $f, g : 2^\omega \to \mathbb{R}$.

Department of Mathematics, University of Connecticut, 341 Mansfield Rd. U-1009, CT 06269, USA
westrick@uconn.edu

KENG MENG NG

A randomness-free characterization of strong jump traceability

joint work with Michael McInerney

The strongly jump traceable degrees have been shown to be very useful, and have been used to illustrate the many connections between algorithmic randomness and classical computability theory. It is a very robust class, with many different characterizations. Unfortunately, all of the characterizations obtained so far have used notions of randomness (either directly or indirectly). Given that the class of strongly jump traceable degrees exhibit strong connections with randomness and computability, it is therefore natural to ask if a degree-theoretic characterization can be obtained for strong jump traceability that does not utilize randomness.

We prove that the strongly jump traceable degrees are exactly those Δ_2^0 degrees that is jump traceable preserving, i.e. a degree $\boldsymbol{a}$ is JT-preserving if $\boldsymbol{a} \cup \boldsymbol{x}$ is jump traceable whenever $\boldsymbol{x}$ is jump traceable. We also give several other degree-theoretic characterizations of strong jump traceability.

Nanyang Technological University, Singapore
kmng@ntu.edu.sg

Todor Tsankov

Universal minimal flows relative to a URS

joint work with Nicolás Matte Bon

A *uniformly recurrent subgroup (URS)* of a locally compact group G is a minimal, conjugation invariant, closed subset of the space of closed subgroups of G. To every minimal action of G, one can naturally associate its stabilizer URS and thus understanding the URSs of a given group gives important information about its non-free, minimal actions. URSs were introduced by Glasner and Weiss and they left open the following basic question: does every URS arise as the stabilizer URS of a minimal action? We answer this question in the affirmative by a universal construction.

Université Paris Diderot
todor@math.univ-paris-diderot.fr

Robin Tucker-Drob

Measure Equivalence, Superrigidity, and Weak Pinsker Entropy

joint work with Lewis Bowen

We show that the class B, of discrete groups which satisfy the conclusion of Popa's Cocycle Superrigidity Theorem for Bernoulli actions, is invariant under measure equivalence. We generalize this to the setting of discrete p.m.p. groupoids, and as a consequence we deduce that any nonamenable lattice in a product of two noncompact, locally compact second countable groups, must belong to B. We also introduce a measure-conjugacy invariant called Weak Pinsker entropy and show that, if G is a group in the class B, then Weak Pinsker entropy is an orbit-equivalence invariant of every essentially free p.m.p. action of G.

Texas A&M Department of Mathematics
rtuckerd@math.tamu.edu

Julien Melleray

Orbit equivalence of Toeplitz flows is Borel complete

A homeomorphism of the Cantor space is *minimal* if all its orbits are dense. Given two minimal homeomorphisms g, h of the Cantor space, one says that g and h are orbit equivalent if there exists a homeomorphism which maps each g-orbit onto an h-orbit.

I will explain why the relation of orbit equivalence of minimal homeomorphisms is Borel complete; more precisely, the relation of orbit equivalence between Toeplitz subshifts is Borel complete, which stands in contrast with the fact that the relation of isomorphism on subshifts is essentially countable.

Perhaps surprisingly, notions from model theory (ultrahomogeneity and Fraïssé limits) play a part in the proof, and I will try to explain why.

Université de Lyon
melleray@math.univ-lyon1.fr

Clinton Conley

Realizing abstract systems of congruences

joint work with Andrew Marks and Spencer Unger

An abstract system of congruences is an equivalence relation on the power set of a finite set meant to encode various possibilities for equidecomposability for some ostensible starting partition. For example, the assertion $\{0\}E\{0, 1, 2\}$ may be read to mean part 0 is equidecomposable with the union of parts $0, 1, 2$. If now some group acts on a space X, we say a partition of X realizes a given abstract system of congruences if the congruence equivalence relation coincides with the induced notion of equidecomposability for the action. Wagon has characterized which such abstract systems of congruences can be realized in the action of rotations of the 2-sphere. We develop an analogous characterization of systems of congruences realized by partitions with the property of Baire, and also investigate some other natural actions.

Carnegie Mellon University
clintonc@andrew.cmu.edu

Gabriel Conant

Pseudofinite groups and tame arithmetic regularity

Arithmetic regularity, developed by Green in 2005, uses discrete Fourier analysis to produce regular decompositions of subsets of finite abelian groups, giving an arithmetic analogue of Szemerédi regularity for finite graphs. More recently, several authors have obtained various "strong arithmetic regularity" results for subsets of finite groups satisfying restricted assumptions. On the combinatorial side, this includes quantitative results assuming stability (Terry-Wolf) or bounded VC-dimension (Alon-Fox-Zhao; Sisask) in finite abelian groups. On the model-theoretic side, in joint work with Pillay and Terry, we give qualitative arithmetic regularity results for finite groups under the same tameness assumptions, by applying local stability and NIP theory to pseudofinite groups. The common ground for these two approaches lies in the study of approximate subgroups, especially the work of Hrushovski and of Breuillard-Green-Tao. In this talk, I will present several of these results, with a focus on the underlying connections between model theory and additive combinatorics.

Department of Mathematics, University of Notre Dame, Notre Dame, IN, USA
gconant@nd.edu

Adrien Deloro

Linearisation in model theory

joint work with Frank Wagner

Model theory deals with algebraic structures in its own way, but the presence of a field is always a good thing. Typically fields come from the very basic Schur lemma in representation theory; of course in model theory one wants definable versions. The most famous result in this vein is Zilber's classical observation that definable fields emerge in many abstract groups of finite Morley rank. But it is not the only such result, nor is finite Morley rank the only model-theoretic framework.

We try to provide the ultimate version of linearisation theorems, in a single statement generalising every Schur-Zilber result known so far and extending them to the natural context of "finite-dimensional theories" (which will be discussed).

Sorbonne Université, Institut de Mathématiques de Jussieu-Paris Rive Gauche, CNRS, Université Paris Diderot. Campus Pierre et Marie Curie, case 247, 4 place Jussieu, 75252 Paris cedex 5, France
adrien.deloro@imj-prg.fr

NADJA HEMPEL

Division Rings with Ranks

joint work with Daniel Palacin

In this talk we analyze division rings which admit a well-behaved ordinal valued rank function on definable sets that behaves like a rudimentary notion of dimension. These are called superrosy division rings. Examples are the quaternions, any superstable division ring (which are known to be algebraically closed fields by theorems of Macintyre[1]/Cherlin-Shelah[2]) and more generally supersimple division rings (which are commutative by a result of Pillay, Scanlon and Wagner[3]). We show that any superrosy division ring has finite dimension over its center, generalizing the aforementioned results [4]. If time permits we will also present some results on division rings of finite burden and weight one [4].

References

[1] ANGUS MACINTYRE, *ω_1-categorical fields*, ***Fundamenta Mathematicae***, vol.70 (1971), no. 3 pp.253–270.

[2] GREGORY CHERLIN, SAHARON SHELAH, *Superstable fields and groups*, ***Annals Math Logic***, vol.18 (1980), pp.227–270.

[3] A. PILLAY, T. SCANLON, F. WAGNER, *Supersimple fields and division rings*, ***Mathematical Research Letters***, vol.5 (1998), pp.473–483.

[4] NADJA HEMPEL, DANIEL PALACIN, *Division rings with ranks*, ***Proceedings of the American Mathematical Society***, vol.146 (2018), no. 2, pp.803–817.

Department of Mathematics, University of California Los Angeles, Box 951555
Los Angeles, CA 90095-1555, USA
nadja@math.ucla.edu

Nicholas Ramsey

Classification theory and the construction of PAC fields

A field K is called *pseudo-algebraically closed* (PAC) if every absolutely irreducible variety defined over K has a K-rational point. These fields were introduced by Ax in his characterization of pseudo-finite fields and have since become an important object of model-theoretic study. A remarkable theorem of Chatzidakis proves that, in a precise sense, independent amalgamation in a PAC field is controlled by independent amalgamation in the absolute Galois group. We will describe how this theorem and a graph-coding construction of Cherlin, van den Dries, and Macintyre may be combined to construct PAC fields with prescribed model-theoretic properties.

Group in Logic, UC Berkeley, Berkeley, CA 94720
nickramsey@berkeley.edu

Nicole Wyatt

Logics and explanations

Logics, it seems, explain things. To take a classic example, Russell deploys first-order logic to explain the behaviour of 'The present king of France is bald'. The debate over the effectiveness of his explanation takes for granted that it is at least possible to use a logic to explain natural language phenomena. But it is a bit of puzzle as to how this works. This talk offers a anti-exceptionalist account of explanation in logic, and sketches some consequences for logical pluralists.

University of Calgary
nicole.wyatt@ucalgary.ca

Simon Hewitt

Some arguments for *Ex Contradictione Quodlibet*

The principle that any proposition whatsoever follows from a contradiction, *Ex Contradictione Quodlibet* (ECN), is enshrined in classical, intuitionistic, and related logics. Although it is frequently encountered as counter-intuitive, and has been questioned by relevance and paraconsistent logicians, little has been done to supply philosophical motivation for ECN.

To the extent that there has been philosophical debate concerning the principle, this has been against the background of an alternative on which some propositions (but not necessarily everthing) follow from a contradiction, as with Priest's **LP** [**?**]. In contrast to this, I explore the prospects for a debate between the proponent of ECN and the historically important view that *no proposition* follows from a contradiction [**?**]. The hope is that this will allow us to get clearer about both ECN and the potential of this alternative (which I call *Ex Contradictione Nihil*).

I lay out four arguments for *ECQ*: C.I. Lewis' 'proof' of the principle (a version of which was discovered by William of Soissons in the 12th century); an argument from Tarski's analysis of consequence; an argument from the necessary truth-preservation, and an argument from the requirements of proof-theoretic harmony. In each case, I diagnose a circularity in the case for ECQ: there is a point at which someone not antecedently disposed to accept ECQ could, and should, object to the argument. This both sheds light on the form a logical theory embodying should take, with respect to structural rules and model-theoretic definitions of consequence, and suggests that if debate is to move beyond *impasse* it will have to take place on a terrain that is somehow more fundamental. I end by suggesting that the theory of meaning is a candidate for such a terrain.

University of Leeds
s.hewitt@leeds.ac.uk

Gil Sagi

Logicality and semantic theory

In this talk we address the question of whether there is a logical consequence relation in natural language, combining our previous work on semantic constraints and recent work by Michael Glanzberg.

In previous work [3], we proposed a model-theoretic framework for logical consequence where there is no strict division of the vocabulary into the logical and the nonlogical terms. The class of models is set by *semantic constraints*: statements in the metalanguage that restrict the interpretation of terms in the language, not necessarily fixing them completely. The framework is a generalization of standard first order logic, and the standard semantic clauses can be reformulated as semantic constraints. In recent work, we consider criteria for logicality of semantic constraints. We generalize the criterion of invariance under isomorphisms for logical terms to apply to semantic constraints. The correct generalization, we claim there, is to the requirement that the class of models satisfying a constraint be closed under isomorphisms.

We apply these results to the question of logic in natural language. In a recent article [1], Michael Glanzberg claims that natural language does not have a logical consequence relation. He argues for this thesis by observing contemporary natural language semantics, and claiming that it cannot distinguish logical consequence from other sorts of entailment. In the first part of the talk, we criticize Glanzberg's arguments, and claim that they do indeed leave room for a logical consequence relation in natural language.

We then discuss another recent article by Glanzberg [2], where he proposes a thesis of partiality in the explanatory force of linguistic theory. He observes that semantic clauses such as:

1. $[\![\text{Ann}]\!] = \text{Ann}$
2. $[\![\text{smokes}]\!] = \lambda x \in D_e.\ x \text{ smokes}$

appear to have only weak explanatory force. Some lexical items, for example predicates, are provided a type by the semantic theory, and while they are also provided with an extension as their semantic value—what their extension is not explained by the theory—but is rather taken for granted. According to Glanzberg, "semantics, narrowly construed as part of our linguistic competence, is only a partial determinant of content." We apply our framework of semantic constraints to isolate the explanatory parts of semantic clauses as those above. We claim that the semantic constraints obtained in this way will be invariant under isomorphisms, in the sense defined before. We thus conclude that semantic theory proper (as delineated by Glanzberg) is a theory of logical consequence in natural language, by the lights of a widely accepted philosophical account of logicality.

References

[1] Michael Glanzberg, *Logical Consequence and Natural Lanaguage*, ***Foundations of Logical Consequence*** (Colin Caret and Ole Hjortland, editors), Oxford University Press, Oxford, 2015, pp. 71–120.

[2] ———, *Explanation and Partiality in Semantic Theory*, ***Metasemantics: New Essays on the Foundations of Meaning*** (Alexis Burgess and Brett Sherman, editors), Oxford University Press, Oxford, 2014, pp. 259–292.

[3] Gil Sagi, *Formality in Logic: From Logical Terms to Semantic Constraints*, ***Logique et Analyse***, vol. 227 (2014), pp. 259-276.

Department of Philosophy, University of Haifa, 199 Aba Khoushy Ave, Mt Carmel, Haifa, Israel
gilisagi@gmail.com

Brice Halimi

Kreisel's problem

Is any logical consequence of ZFC ensured to be true ?(KP) Kreisel and Boolos both proposed an answer to (KP), taking "truth" to mean truth in the background set-theoretic universe. My talk will advocate another answer, which lies at the level of the models of ZFC, so that "truth" remains the usual semantic notion. This other, model-scaled answer relies on the fact that any model of ZFC can be shown to contain "internal models," and thus can be compared to a particular background universe itself.

After defining logical consequence w.r.t. any given model of ZFC, I will set out the answer to the original problem (KP) prompted by this definition, and present further results bearing on internal models. Finally, the semantics interpreting internal models as accessible worlds leads to an "internal modal logic" in which internal reflection is shown to correspond to modal reflexivity, and resplendency to the modal axiom **4**.

Department of Philosophy, Université Paris Nanterre, Nanterre, France
bhalimi@parisnanterre.fr

Alessandra Palmigiano

Constructive Canonicity of Inductive Inequalities

This talk, based on [8], discusses the canonicity of inductive inequalities in a constructive meta-theory, for classes of logics algebraically captured by varieties of normal and regular lattice expansions. These results are obtained using the tools of *unified correspondence theory* [6, 10, 7, 9, 3, 2, 13], and contribute to develop a theoretical environment in which different proof techniques for canonicity and correspondence can be systematically compared and connected to each other (cf. [12, 16, 11, 17]). These canonicity results contribute to applications of semantic results to proof-theoretic issues [14, 1, 15] but also of logic to social sciences [5, 4].

References

[1] M. Bílková, G. Greco, A. Palmigiano, A. Tzimoulis, N. Wijnberg. The logic of resources and capabilities. In *The Review of Symbolic Logic*, 1–40, 2018.

[2] W. Conradie, A. Craig, A. Palmigiano, Z. Zhao. Constructive canonicity for lattice-based fixed point logics. In *Proc. WoLLIC 2017*, pages 92–109. Springer, 2017.

[3] W. Conradie, Y. Fomatati, A. Palmigiano, S. Sourabh. Algorithmic correspondence for intuitionistic modal mu-calculus. *Theor. Comp. Sci.*, 564:30–62, 2015.

[4] W. Conradie, S. Frittella, A. Palmigiano, M. Piazzai, A. Tzimoulis, N. Wijnberg. Toward an epistemic-logical theory of categorization. In *Proceedings TARK 2017, Electronic Proceedings in Theoretical Computer Science* 251, pages 170–189.

[5] W. Conradie, S. Frittella, A. Palmigiano, M. Piazzai, A. Tzimoulis, N. Wijnberg. Categories: How I Learned to Stop Worrying and Love Two Sorts. In *Proc. WoLLIC 2016*, pages 145–164. Springer, 2016. ArXiv preprint 1604.00777.

[6] W. Conradie, S. Ghilardi, A. Palmigiano. Unified correspondence. In A. Baltag and S. Smets, editors, *Johan van Benthem on Logic and Information Dynamics*, volume 5 of *Outstanding Contributions to Logic*, pages 933–975. Springer, 2014.

[7] W. Conradie and A. Palmigiano. Algorithmic correspondence and canonicity for distributive modal logic. *Annals of Pure and Applied Logic*, 163(3):338 – 376, 2012.

[8] W. Conradie and A. Palmigiano. Constructive canonicity of inductive inequalities. Submitted. ArXiv preprint 1603.08341.

[9] W. Conradie and A. Palmigiano. Algorithmic correspondence and canonicity for non-distributive logics. Submitted, arXiv:1603.08515.

[10] W. Conradie, A. Palmigiano, S. Sourabh. Algebraic modal correspondence: Sahlqvist and beyond. *Journal of Logical and Algebraic Methods in Programming*, 91:60–84, 2017.

[11] W. Conradie, A. Palmigiano, S. Sourabh, Z. Zhao. Canonicity and relativized canonicity via pseudo-correspondence: an application of ALBA. Submitted. ArXiv preprint 1511.04271.

[12] W. Conradie, A. Palmigiano, Z. Zhao. Sahlqvist via translation. Submitted.

[13] S. Frittella, A. Palmigiano, L. Santocanale. Dual characterizations for finite lattices via correspondence theory for monotone modal logic. *Journal of Logic and Computation*, 27(3):639–678, 2017.

[14] G. Greco, M. Ma, A. Palmigiano, A. Tzimoulis, Z. Zhao. Unified correspondence as a proof-theoretic tool. *Journal of Logic and Computation*, 2016. doi: 10.1093/logcom/exw022. ArXiv preprint 1603.08204.

[15] G. Greco, A. Palmigiano. Lattice logic properly displayed *Lecture Notes in Computer Science* 10388 (2017) p. 153–169.

[16] A. Palmigiano, S. Sourabh, Z. Zhao. Jónsson-style canonicity for ALBA-inequalities. *Journal of Logic and Computation*, 27(3):817–865, 2017.

[17] A. Palmigiano, S. Sourabh, Z. Zhao. Sahlqvist theory for impossible worlds. *Journal of Logic and Computation*, 27(3):775–816, 2017.

Delft University of Technology
a.palmigiano@tudelft.nl

Chuangjie Xu

Unifying functional interpretations of nonstandard/uniform arithmetic

We extend Oliva's method [3] to develop a parametrised functional interpretation for nonstandard arithmetic. By instantiating the parametrised relations, we obtain the Herbrand functional interpretations introduced in [2, 4] for nonstandard arithmetic as well as the usual, well-known ones such as modified realisability for Berger's uniform Heyting arithmetic [1]. We implement it in the Agda proof assistant by making use of Agda's parameterised modules. This allows us to extract computer programs from proofs in the nonstandard/uniform arithmetic via different instantiations of our unifying functional interpretation.

References

[1] Ulrich Berger, *Uniform Heyting arithmetic*, ***Annals of Pure and Applied Logic***, vol. 133 (2005), no. 1, pp. 125–148.

[2] Fernando Ferreira and Jaime Gaspar, *Nonstandardness and the bounded functional interpretation*, ***Annals of Pure and Applied Logic***, vol. 166 (2015), no. 6, pp. 701–712.

[3] Paulo Oliva, *Unifying functional interpretations*, ***Notre Dame Journal of Formal Logic***, vol. 47 (2006), no. 2, pp. 263–290.

[4] Benno van den Berg, Eyvind Briseid, and Pavol Safari, *A functional interpretation for nonstandard arithmetic*, ***Annals of Pure and Applied Logic***, vol. 163 (2012), no. 12, pp. 1962–1994.

Ludwig-Maximilians-Universität München
xu@math.lmu.de

Martín Hötzel Escardó

Univalent mathematics at work

I will illustrate how to work in Voevodsky's univalent mathematics in practice, both on paper, informally, and in (cubical) Agda, formally, with examples from my recent work on constructive mathematics.

School of Computer Science, University of Birmingham
m.escardo@cs.bham.ac.uk

Ryota Akiyoshi

"Proofs as Programs" Revisited

Schwichtenberg has developed the program called "Proofs as Programs" by measuring the "complexity as programs" of proofs in an arithmetical system (with recursion) in [7]. Technically, he used Arai's observation in [4] to use a slow growing hierarchy $G_a(\cdot)$ in order to "clime down" tree ordinals due to Buchholz [5]. We remark that this is going back to Girard's hierarchy comparison theorem [6].

In this talk, we sketch another approach to this program by focusing on parameter-free subsystems of Girard's System F. There are at least two advantages in this approach. (i) It is simpler and smoother than the original one. (ii) It is relatively easy and uniform to extend our results to stronger fragments (corresponding to theories of iterated inductive definitions based on intuitionistic logic).

For explaining basic ideas, we focus on the weakest parameter-free fragment of System F. In this fragment called $\mathbb{F}_0$, a second-order type $\forall\alpha.A$ is permitted only if A is $\forall$-free and contains no other variable than X.

The upperbound of the complexity of terms in $\mathbb{F}_0$ is computed as follows. For a finite term M in $\mathbb{F}_0$, we define the relation $\vdash^{\alpha}_{m} M : A$ saying that (i) the upperbound of the complexity of M is measured by a tree ordinal α, (ii) the "cut-rule" of the rank $< m$ is sufficient. In particular, we introduce a miniaturized version of the Ω-rule due to Buchholz [5]. Next, we prove theorems for the relation $\vdash^{\alpha}_{m} M : A$ corresponding to the predicative cut-elimination and impredicative cut-elimination theorems in proof-theory.

Now, the upperbound theorem is stated as follows:

Theorem 1. *Let f be a representable function in $\mathbb{F}_0$ with $M : \boldsymbol{N} \Rightarrow \boldsymbol{N}$. Then, $\models^{D_0(d\times(n+1))}_{0} \mathcal{D}_0\mathcal{D}^m_1(M\mathsf{S}^n 0) : \boldsymbol{N}$ with $d = D^m_1(\Omega\times m)$ for some $m \geq 4$. Therefore, there is m such that for all $n \geq m \geq 4$*

$$|\mathcal{D}_0\mathcal{D}^m_1(M\mathsf{S}^n 0)| < G_{D_0 D_1^{m+2}0}(n).$$

(Here, D_1 is the standard exponential function with the base ω, and D_0 is the collapsing function from Ω_2 into Ω_1.)

The lowerbound is proved by using Schwichtenberg's result [7] and Aehlig's one [1]:

Theorem 2. *The function expressed by the formula $\forall x\exists y(D_0 D^m_1 0)[x]^y = 0$ in arithmetic is representable in $\mathbb{F}_0$.*

If time is permitting, we sketch how to extend our approach into stronger parameter-free fragments of System F by climbing up the hierarchy defined by Aehlig [1].

This work is partially supported by JSPS KAKENHI 16K16690 and 17H02265.

References

[1] Klaus Aehlig, *Parameter-free polymorphic types*, ***Annals of Pure and Applied Logic***, 156(1): 3–12, 2008.

[2] Ryota Akiyoshi, *An Ordinal-Free Proof of the Complete Cut-Elimination Theorem for* Π^1_1*-CA + BI with the* ω*-rule*, ***The Mints' memorial issue of the IfCoLog Journal of Logics and their Applications***, 4(4): 867–884, 2017.

[3] Ryota Akiyoshi and Kazushige Terui, *Strong normalization for the parameter-free polymorphic lambda calculus based on the Omega-rule*, ***First International Conference on Formal Structures for Computation and Deduction (FSCD)***, Leibniz International Proceedings in Informatics, 5:1–5:15, 2016.

[4] Toshiyasu Arai, *A slow growing analogue to Buchholz' proof*, ***Annals of Pure and Applied Logic***, 54(2): 101–120, 1991.

[5] Wilfried Buchholz, *An independence result for* $(\Pi^1_1\text{-}CA) + BI$, ***Annals of Pure and Applied Logic***, 33, 131–155, 1987.

[6] Jean-Yves Girard, Π^1_2*-Logic, Part 1: Dilators*, ***Annals of Mathematical Logic***, 21(2-3):75–219, 1981.

[7] Helmut Schwichtenberg, *Proofs as Programs*, ***Proof theory: a selection of papers from the Leeds Proof Theory Programme 1990*** (P. Aczel and H. Simmons, editors), 81–113, Cambridge University Press, 1990.

Waseda Institute for Advanced Study, Tokyo, Nishi Waseda 1-6-1, Japan
Keio University, Tokyo, Mita 2-15-45, Japan
georg.logic@gmail.com

Agi Kurucz

Horn fragments of the Halpern-Shoham interval temporal logic $\mathcal{HS}$

The elegance and expressive power of $\mathcal{HS}$ have attracted the attention of the temporal and modal logic communities, as well as many other areas of computer science, AI, philosophy and linguistics. As $\mathcal{HS}$-satisfiability over various timelines is undecidable [1], the quest for 'tame' fragments began in the 2000s, and have resulted in a substantial body of literature that identified a number of ways of reducing its expressive power. We discuss recent results on the computational complexity of Horn and core fragments of $\mathcal{HS}$ [2].

References

[1] J. Halpern and Y. Shoham, *A propositional modal logic of time intervals,* **Journal of the ACM** 38 (4), 935–962, 1991.

[2] D. Bresolin, A. Kurucz, E. Muñoz-Velasco, V. Ryzhikov, G. Sciavicco and M. Zakharyaschev, *Horn fragments of the Halpern-Shoham interval temporal logic,* **ACM Transactions on Computational Logic** 18(3), 22:1-22:39, 2017.

Department of Informatics, King's College London, Strand Campus, Bush House, 30 Aldwych, London WC2B 4BG, United Kingdom
agi.kurucz@kcl.ac.uk

Daniele Mundici

Łukasiewicz logic: 98 years

After almost a century since its origination, Łukasiewicz logic is still a vibrant research topic. A recent development is the differential semantics of Łukasiewicz infinite-valued propositional logic L_∞, according to which a formula θ follows from a set Θ of premises if, and only if, the following two conditions are satisfied:

1. Every model m of Θ is a model of θ (as per the Bolzano-Tarski paradigm), and
2. Any infinitesimal perturbation $m + \mathrm{d}m$ which is a model of Θ is a model of θ.

Condition 2 makes sense because the space of models is endowed with the topological-differential structure of a Tychonoff cube. This notion of semantic consequence turns out to be equivalent to the classical notion of syntactic consequence—stating that θ is derivable by finitely many applications of modus ponens to a finite subset of ($\Theta\cup$ tautologies of L_∞). This is the completeness theorem for L_∞. If Θ is finite, Condition 2 automatically follows from Condition 1 (Hay's theorem). If Θ is infinite this is no longer true in general (Wójcicki's theorem).

References

[1] R. Cignoli, I.M.L.D'Ottaviano, D.Mundici, **Algebraic foundations of many-valued reasoning,** Trends in Logic, Vol. 7, Kluwer Academic Publishers, Dordrecht, 2000.

[2] L.S. Hay, *Axiomatization of the infinite-valued predicate calculus*, **Journal of Symbolic Logic**, Vol. 28 (1963), pp. 77-86.

[3] D.Mundici, **Advanced Łukasiewicz calculus and MV-algebras,** Trends in Logic, Vol. 35, Springer, New York, 2011.

[4] D.Mundici, *The differential semantics of Łukasiewicz syntactic consequence*, Chapter 7, In: **Petr Hájek on mathematical fuzzy logic,** (F. Montagna, Editor), Outstanding Contributions, Vol. 6, Springer International Publishing Switzerland, 2015, pp.143-157.

[5] R. Wójcicki, **Theory of Logical Calculi: Basic Theory of Consequence Operations**, Synthese Library, Vol. 199, Kluwer, Dordrecht, 1988.

Dept. of Mathematics and Computer Science, University of Florence,
Viale Morgagni 67, 50134 Florence, Italy
mundici@math.unifi.it

Paritosh K. Pandya

Expressive-completeness and Decidability of Metric Temporal Logics: Recent Progress

The celebrated Kamp Theorem established Expressive completeness of Linear Temoral Logic (LTL) with respect to $FO[<]$, the first order theory of words. The equally celebrated Buchi theorem established (effective) expressive completeness of Monadic Second Order Logic $MSO[<]$ with respect to deterministic finite state automata (DFA). The later also provides a method for deciding satisfiability and model checking of $MSO[<]$ formulae using algorithms over DFA.

Generalising this approach to real-time logics has been problematic. In this talk, we will survey some recent progress in formulating expressively complete real-time logics. Recent results of Hunter, Ouaknine and Worrell [4, 3] have led to metric temporal logics which are expressively complete for $FO[<, +1]$, the first order tgheory of timed words with metric distance predicate +1. However, these logics have undecidable satisfiability.

Towards the quest for expressive and decidable real-time logics, we will survey past work on logics [1, 7, 2] which explored expressive completeness with Non-deterministic timed automata (NTA). We will then look at recent attempts at extending Metric Temporal Logic to get expressive completeness with 1-clock Alternating Timed Automata (*1ATA*). A monadic second order logic with guarded metric quantifiers, $QkMSO$, has decidable satisfiability and model checking. This logic will be presented and its expressive power will be characterised using a subclass of *1ATA* as well as a metric temporal logic *RegMTL*. Logic *RegMTL* (see [6]) extends the well-known $MTL[U_I]$ [5] with a regular expression modality.

References

[1] R. Alur, T. Feder, and T. Henzinger. The benefits of relaxing punctuality. *J.ACM*, 43(1):116–146, 1996.

[2] Y. Hirshfeld, and A. Rabinovich. An expressive temporal logic for real time. In *MFCS*, pages 492–504, 2006.

[3] P. Hunter. When is metric temporal logic expressively complete? In *CSL*, pages 380–394, 2013.

[4] P. Hunter, J. Ouaknine, and J. Worrell. Expressive completeness for metric temporal logic. In *LICS*, pages 349–357, 2013.

[5] J. Ouaknine, and J. Worrell. On the decidability of metric temporal logic. In *LICS*, pages 188–197, 2005.

[6] S. Krishna, K. Madnani, and P. K. Pandya. Making Metric Temporal Logic Rational. In *MFCS 2017*, pages 77:1–77:14, 2017.

[7] T. Wilke. Specifying timed state sequences in powerful decidable logics and timed automata. In *FTRTFT*, pages 694–715, 1994.

[8] S. Krishna, K. Madnani, and P.K. Pandya, Büchi-Kamp Theorems for 1-clock ATA, In *CoRR*, abs/1802.02514, 2018.

Tata Institute of Fundamental Research, Mumbai 400005, India
pandya@tifr.res.in

Amanda Vidal

Many-valued modal logics: axiomatizability issues

Modal logic is one of the most developed and studied non-classical logics, yielding a beautiful equilibrium between complexity and expressibility. Generalizations of the concepts of necessity and possibility offer a rich setting to model and study notions from many different areas, including proof-theory, temporal and epistemic concepts, workflow in software applications, etc. On the other hand, substructural logics provide a formal framework to manage vague and resource sensitive information in a very general (and so, adaptable) fashion. *Many-valued modal logics* is a field in ongoing development. While the first publications on the topic can be traced back to the 90s [5, 6], it has been only in the latter years when a more systematic work has been developed, addressing the axiomatizability question in general, characterization and study of the model-theoretic notions analogous to the ones from the classical case, decidability and applicability issues, etc (see eg. [7], [3, 4], [1], [9], [8], [2]...).

In this talk we present some recent results for these logics, focused on their decidability and axiomatizability. In particular, we exhibit a large family of many-valued modal logics, including several very natural cases (including modal expansions of Łukasiewicz and Product logics, two of the three main BL extensions) that even if they enjoying the finite model property, are undecidable. Thus, no R.E axiomatization can exist for them, in contrast to what happens with their propositional counterparts. We show that simple properties in the underlying class of algebras (non-contractivity of the strong conjunction operation, and a technical but easy notion related to the existence and behavior of the infimum of certain values) allow the construction of a set of formulas capturing the Post correspondence Problem in a large family of modal logics. In the particular case of Łukasiewicz logic, this allow us, together with completeness with respect to so-called witnessed models (where, in contrast to other many-valued modal logics, each modal formula is evaluated as that formula in some successor world) to also show that the usually called global Łukasiewic modal logic cannot be axiomatized.

References

[1] F. Bou, F. Esteva, L. Godo, and R. Rodríguez, *On the minimum many-valued modal logic over a finite residuated lattice*, ***Journal of Logic and Computation***, vol. 21 (2011), no. 5, pp. 739–790.

[2] X. Caicedo, G. Metcalfe, R. Rodríguez, and J. Rogger, *A finite model property for Gödel modal logics*, ***WOLLIC 2013, Lecture Notes in Computer Science***, (In L. Libkin, et.al eds.), vol. 8071, Springer (2013), pp. 226–237.

[3] X. Caicedo and R. O. Rodriguez, *Standard Gödel modal logics*, ***Studia Logica***, vol. 94 (2010), no. 2, pp. 189–214.

[4] ——— *Bi-modal Gödel logic over* $[0,1]$*-valued kripke frames*, ***Journal of Logic and Computation***, vol. 25 (2015), no. 1, pp. 37–55.

[5] M. Fitting, *Many-valued modal logics*, ***Fundamenta Informaticae***, vol. 15(1992), pp. 235–254.

[6] ——— *Many-valued modal logics II*, ***Fundamenta Informaticae***, vol. 17 (1992), pp. 55–73.

[7] G. Hansoul and B. Teheux, *Extending Łukasiewicz logics with a modality: Algebraic approach to relational semantics*, ***Studia Logica***, vol. 101 (2013), no. 3, pp. 505–545.

[8] G. Metcalfe and N. Olivetti, *Towards a proof theory of Gödel modal logics*, ***Logical Methods in Computer Science***, vol. 7 (2011), no. 2, 27.

[9] A. Vidal, F. Esteva, and L. Godo, *On modal extensions of product fuzzy logic*, ***Journal of Logic and Computation***, vol. 27(2017), no. 1, pp. 299–336.

Institute of Computer Science, Czech Academy of Sciences
amanda@cs.cas.cz

Contributed Papers

The following pages reflect the program as planned at mid-June. See https://lc18.uniud.it/program/ for possible changes in the program.

Svetlana Aleksandrova

Σ−definability of hereditarily finite and hereditarily finite list superstructures

The notion of Σ−definability of structures in admissible sets was introduced by Ershov [1]. Later it was developed into several suchlike notions, different in the power of preservation of desirable structure properties, such as, for example, Σ−theory of a structure.

In this talk we will discuss properties of hereditarily finite $\mathbb{HF}(\mathfrak{M})$ and hereditarily finite list superstructures $\mathbb{HW}(\mathfrak{M})$ with regard to this Σ−definability notions. (See [1], [2] for the structures definitions). In particular, we prove that, for a given structure $\mathfrak{M}$, hereditarily finite superstructure $\mathbb{HF}(\mathfrak{M})$ is Σ−definable in a hereditarily finite list superstructure $\mathbb{HW}(\mathfrak{M})$ and vice versa.

We will also discuss some corollaries regarding nuanced relationships between $\mathbb{HF}(\mathfrak{M})$ and $\mathbb{HW}(\mathfrak{M})$.

References

[1] Ershov, Yu. L., ***Definability and Computability***, Consultants Bureau, New York-London-Moscow, 1996.

[2] Goncharov, S. S., Sviridenko, D. I., *Σ−programming (Russian)*, ***Vych. Sist.***, (1985), no. 107, pp. 3–29.

Novosibirsk State University, Novosibirsk, Russia
svet-ka@eml.ru

Pavel Arazim

Do deviant logicians speak a different language? And does it matter?

Discussions about plurality of logics and cogency of logical pluralism have very soon stumbled upon a curious thesis which with some variations on the same theme keeps reappearing till today. The thesis says that in fact no real disagreement concerning logic is possible because differing in acceptance or refusal of logical laws ammounts to endowing at least a part of logical vocabulary with a different meaning and therewith talking about something else rather than contradicting the other party. This thesis was advocated by Carnap and Quine and there are contemporary authors sympathetic with it, for instance Kissel and Warren. Both Carnap and Quine seem to perceive this thesis as close to self-obvious, as they are content with rather sketchy arguments in its defence. Yet the curious fact about the thesis is that Carnap uses it as a support for logical pluralism, while Quine as a support for logical monism. At least prima facie this thesis leads to an antinomy which forces us to doubt it and examine arguments for and against it more cautiously. We will show that this thesis is in conflict with pragmatism and holism present in Quine and at least implicit in Carnap. Ultimately it cannot be a meaningful thesis or it cannot play a fundamental role in debates over the cogency of logical pluralism. Deciding to what degree a given question concerns meanings of given terms or something else, typically matters of fact, does not determine to what degree it is a genuine question. It will be shown how this entails that logical pluralism and logical monism, as standardly conceived, either are not cogent theses or are not contradictory to each other, so that we can unite both in a single account of the plurality of logics I prose and call logical dynamism.

References

[1] Carnap, Rudolf, ***Logische Suntax der Sprache***, Wien: Springer, 1934.

[2] Kissel, Teresa Kouri, *Logical Pluralism from a Pragmatic Perspective*, ***Australasian Journal of Philosophy***, forthcoming.

[3] Quine, Willard Van Orman, ***Philosophy of Logic***, Cambridge: Harvard University Press, 1986.

[4] Warren, Jared, *Change of Logic, Change of Meaning*, ***Philosophy and Phenomenological Research***, vol. XCVI (2018), no. 2, pp. 421–442.

Department of logic, Philosophical Institute of Czech Academy of Sciences, Jilska 1, Praha, Czech Republic
arazim@flu.cas.cz

Piotr Błaszczyk

Trends in the history of infinity

Cantor established two kinds of infinity: cardinal and ordinal numbers, each with its own arithmetic and its own relation *greater than*. In modern developments, ordinal numbers are special sets, cardinal numbers are specific ordinal numbers. In both cases, the set of natural numbers $\mathbb{N}$ makes the yardstick of infinity – be it the cardinal number $\aleph_0$ or the ordinal ω. However, while Cantor infinities try to extend the arithmetic of finite numbers, the addition and multiplication of ordinal numbers are not commutative. Moreover, while there are many possible well-orderings on the set $\mathbb{N}$, Cantor considered the *natural* one; in [2] he also considered the *natural* order of the real numbers. Cantor could never explain what does mean *natural* order in mathematical terms.

In [4] Euler introduced numbers that exceed any finite number. Still, while in his development finite numbers form an ordered field, Euler infinite numbers also belong to the ordered field. Consequently, when N is infinite, so is $N - 1$ and $N/2$.

Thus, Cantor's and Euler's investigations exemplify competing trends in the history of infinity founded on set theory or algebra.

We will argue that the theory of surreal numbers developed in [3] provides a uniform perspective that allows one to compare these two trends. We will argue that the perspective of ordered fields provides a more general and consistent account of infinity. (a) The theory of real closed fields developed in [1] provides mathematical reasons to treat a total order as a *natural* one. Namely, in some fields, e.g. in the field of surreal numbers, there is only one total order compatible with addition and multiplication. (b) Surreal numbers include ordinal numbers. (c) The addition and multiplication of ordinal numbers is commutative when they are taken as surreals. (d) There exist negative and fractional ordinal numbers, when ordinal numbers are considered as elements of the filed of surreals.

References

[1] Emil Artin, Otto Schreier, *Algebraische Konstruktion reeller Korper*, ***Abhandlungen aus dem Mathematischen Seminar der Hamburgischen Universität***, vol. 5 (1926), pp. 85–99.

[2] Georg Cantor, *Beiträge zur Begründung der transfiniten Mengenlehre. Der Ordnungstypus θ des Linearkontinuums*, ***Mathematische Annalen***, vol. 46 (1895), pp. 481–512.

[3] John Conway, ***On Numbers and Games***, AK Peters, 2001.

[4] Leohnard Euler, ***Introductio in analysin infinitorum***, Bousquet, 1748.

Institute of Mathematics, Pedagogical University of Cracow, Kraków, Podchorążych 2, Poland
pb@up.krakow.pl

Silvia Barbina

The theory of Steiner triple systems

Finite Steiner triple systems (STSs) are well known combinatorial objects for which the literature is extensive. An STS is a set S together with a collection $\mathcal{B}$ of subsets of S of size 3 such that any two elements of S belong to exactly one element of $\mathcal{B}$. The existence of the Fraïssé limit M_{F} of all finite STSs is known, but no description of its theory was available so far. In joint work with Enrique Casanovas, we describe the theory of M_{F}, we prove that it has quantifier elimination and that it has no countable saturated model.

Open University
silvia.barbina@open.ac.uk

Giulia Battilotti

Reading Bi-logic in first order language

joint work with Milos Borozan and Rosapia Lauro Grotto

We see how one can formalise the main aspects of Matte Blanco's Bi-logic [1] in first order language [2, 3, 4]. In particular, the formalisation allows easily to read also Freud's original way to distinguish between "word presentation" and "thing presentation", introduced in [5], that is at the root of his theory, in the distinction between closed terms and variables, that had been introduced short before in formal logic. Moreover, we suggest how the psychoanalytical problems so involved could be discussed in terms of the modal logic S4 and in terms of linear logic, both of which are aimed to overcome the limitations of first order logic, due to the formal notion of term. Then we think that logic should consider Freud's view of the problem, and, in general, that the particular clustering of the logical notions induced by the reading of the psychoanalytical notions, and conversely, can offer a new opportunity for both subjects.

References

[1] I. Matte Blanco, ***The unconscious as infinite sets***, Duckworth, London 1975.

[2] G. Battilotti , *Symmetry vs. Duality in Logic: an interpretation of Bi-logic to model cognitive processes beyond inference* , ***International Journal of Cognitive Informatics and Natural Intelligence 8 (2014) 83-97.***, vol. 8 (2014), pp. 83–97.

[3] G. Battilotti, M. Borozan, R. Lauro Grotto, *Bi-logic and first order language*, ***In preparation.***

[4] Milos Borozan, ***When the possible becomes necessary: towards a formalisation of Matte Blanco's Bi-logic (Italian)***, Master Thesis, University of Florence, R. Lauro Grotto advisor and G. Battilotti co-advisor, 2017.

[5] S. Freud, ***Zur Auffassung der Aphasien. Eine kritische Studie***, Franz Deutictke, Leipzig und Wien, 1891.

Dept. of Mathematics - University of Padua
giulia@math.unipd.it

Nikolay Bazhenov

Categoricity spectra for linear orders

Let $\mathbf{d}$ be a Turing degree. A computable structure $\mathcal{S}$ is $\mathbf{d}$-*computably categorical* if for any computable copy $\mathcal{A}$ of $\mathcal{S}$, there is a $\mathbf{d}$-computable isomorphism from $\mathcal{A}$ onto $\mathcal{S}$. The *categoricity spectrum* of $\mathcal{S}$ is the set

$$CatSpec(\mathcal{S}) = \{\mathbf{d} : \mathcal{S} \text{ is } \mathbf{d}\text{-computably categorical}\}.$$

A degree $\mathbf{c}$ is the *degree of categoricity* of $\mathcal{S}$ if $\mathbf{c}$ is the least degree in the spectrum $CatSpec(\mathcal{S})$. R. Miller [1] constructed the first example of a computable structure with no degree of categoricity.

In this work, we build a new series of computable linear orders with no degree of categoricity: For every computable successor ordinal $\alpha \geq 4$, the set of PA degrees over $\mathbf{0}^{(\alpha)}$ is the categoricity spectrum for a scattered linear order.

Suppose that $\mathbf{d}$ is the degree of categoricity for a computable structure $\mathcal{S}$. The degree $\mathbf{d}$ is called the *strong degree of categoricity* for $\mathcal{S}$ if it satisfies the following:

(∗) There are two computable copies $\mathcal{A}$ and $\mathcal{B}$ of $\mathcal{S}$ such that every isomorphism from $\mathcal{A}$ onto $\mathcal{B}$ computes $\mathbf{d}$.

We say that $\mathbf{d}$ is the *non-strong degree of categoricity* for $\mathcal{S}$ if it does not satisfy $(*)$. The first examples of structures with non-strong degrees of categoricity were independently built in [2] and [3].

Here we prove the following: If α is a computable infinite ordinal, then there is a computable scattered linear order such that it is $\Delta^0_{\alpha+2}$ categorical, not Δ^0_α categorical, and has non-strong degree of categoricity.

References

[1] R. Miller, **d**-*computable categoricity for algebraic fields*, ***Journal of Symbolic Logic***, vol. 74 (2009), no. 4, pp. 1325–1351.

[2] N. A. Bazhenov, I. Sh. Kalimullin, and M. M. Yamaleev, *Degrees of categoricity vs. strong degrees of categoricity*, ***Algebra and Logic***, vol. 55 (2016), no. 2, pp. 173–177.

[3] B. F. Csima and J. Stephenson, *Finite computable dimension and degrees of categoricity*, ***preprint***.

Sobolev Institute of Mathematics, 4 Acad. Koptyug Ave., Novosibirsk, Russia;
Novosibirsk State University, 2 Pirogova St., Novosibirsk, Russia
bazhenov@math.nsc.ru

Stefania Boffa

Epistemic logic of sequences of partitions

joint work with Brunella Gerla and Ricardo O. Rodriguez

In former papers we gave a representation of algebraic structures related with many-valued logics such as Kleene algebras and IUML-algebras as sequences of rough sets, i.e., increasingly accurate approximations of sets. In order to do that, we started from sequences of partitions of a given universe, in such a way that each partition refines the partition before it. Further, we allow the case that in the refinement sequence some of the elements of the starting universe can get lost.

Here we present a modal system, called $OPS5$, whose semantics consists of such sequences of partitions. The language of $OPS5$ contains two families of modal operators $\Box_i$ and $\rhd_i$ indexed in $I = \{1, \dots, n\}$. An OPK-model is a triple $\mathcal{M} = (U, \{R_i\}_{i \in I}, \pi)$, where U is the set of worlds, π is an evaluation of variables in each world and for every $i \in I$, R_i is an equivalence relation on subsets of U, such that $R_j(u) \subseteq R_i(u)$, for each $i < j$ and $u \in U$. A formula $\Box_i\varphi$ is true in a world v if the formula φ is true in all worlds equivalent to v wrt R_i. A formula $\triangleright_i\varphi$ is true in a world v if φ is true in v and $R_i(v)$ is not a singleton.

Further, we consider $OPS5$ as an epistemic logic for dealing with the knowledge of an agent k during a sequence $(t_1, \dots, t_n)$ of instants of time. Fixing an OPK-model, if the equivalence class of a world is a singleton then we consider that the agent is not interested in that evaluation at that moment. Hence if φ is a formula of $OPS5$ and $i \in \{1, \dots, n\}$, the statement $\Box_i\varphi$ is interpreted as "k knows φ at time t_i"and the statement $\triangleright_i\varphi$ is interpreted as "φ is true and k is interested to know it at time t_i".

Università dell' Insubria (Italy)
boffa@uninsubria.it

Marija Boričić

Soundness and completeness of a high probabilities sequent calculus

We present a system **LKprob**(ε) (see [1], [3] and [4]) making it possible to work with expressions of the form $\Gamma \vdash^n \Delta$, a generalization of Gentzen's sequents $\Gamma \vdash \Delta$ of classical propositional logic **LK**, with the intended meaning that 'the probability of the sequent $\Gamma \vdash \Delta$ is greater than or equal to $1 - n\varepsilon$', for a given small real $\varepsilon > 0$ and any natural number n (see [6] and [7]). For instance, **LKprob**(ε) is based on rules of the following form:

$$\frac{\Gamma A B \vdash^n \Delta}{\Gamma A \wedge B \vdash^n \Delta}(\wedge\vdash) \quad \frac{\Gamma \vdash^n A\Delta \quad \Gamma \vdash^m B\Delta}{\Gamma \vdash^{m+n} A \wedge B\Delta}(\vdash\wedge)$$

A model for **LKprob**(ε) is a mapping $p : \mathrm{Seq} \to I \cap [0, 1]$, where $I = \{1 - n\varepsilon | n \in \mathbf{N}\}$, satisfying the following conditions: *(i)* $p(A \vdash A) = 1$, for any formula A; *(ii)* if $p(AB \vdash) = 1$, then $p(\vdash AB) = p(\vdash A) + p(\vdash B)$, for any formulae A and B; *(iii)* if sequents $\Gamma \vdash \Delta$ and $\Pi \vdash \Lambda$ are equivalent in **LK**, in sense that there are proofs for both sequents $\bigwedge\Gamma \to \bigvee\Delta \vdash \bigwedge\Pi \to \bigvee\Lambda$ and $\bigwedge\Pi \to \bigvee\Lambda \vdash \bigwedge\Gamma \to \bigvee\Delta$ in **LK**, then $p(\Gamma \vdash \Delta) = p(\Pi \vdash \Lambda)$.

A theory **LKprob**$(\varepsilon)(\sigma_1, \dots, \sigma_n)$, an extension of **LKprob**(ε) by the list of new axioms $\sigma_1, \dots, \sigma_n$, is said to be consistent iff there exists a sequent $\Gamma \vdash^0 \Delta$ which is unprovable in **LKprob**$(\varepsilon)(\sigma_1, \dots, \sigma_n)$. We prove that each consistent theory can be extended to a maximal consistent theory, and as a consequence, soundness and completeness is also proved. The system **LKprob**(ε) can be considered a particular case of our general approach to probabilization of sequent calculus (see [1], [2] and [5]).

References

[1] M. Boričić, *Hypothetical syllogism rule probabilized*, ***Bulletin of Symbolic Logic***, Vol. 20 (3) (2014), pp. 401–402, (Abstract), Logic Colloquium 2012.

[2] M. Boričić, *Models for the probabilistic sequent calculus*, ***Bulletin of Symbolic Logic***, Vol. 21 (1) (2015), p. 60, (Abstract), Logic Colloquium 2014.

[3] M. Boričić, *Suppes–style rules for probability logic*, ***Bulletin of Symbolic Logic***, Vol. 22 (3) (2016), p. 431 (Abstract), Logic Colloquium 2015.

[4] M. Boričić, *Suppes–style sequent calculus for probability logic*, ***Journal of Logic and Computation***, Vol. 27 (4) (2017), pp. 1157–1168.

[5] M. Boričić, *Sequent calculus for classical logic probabilized*, ***Archive for Mathematical Logic*** (to appear)

[6] P. Suppes, *Probabilistic inference and the concept of total evidence*, ***Aspects of Inductive Inference***, (J. Hintikka and P. Suppes, editors), North–Holland, Amsterdam, 1966, pp. 49–55.

[7] C. G. Wagner, *Modus tollens probabilized*, ***British Journal for the Philosophy of Science***, vol. 54 (4) (2004), pp. 747–753.

Faculty of Organizational Sciences, University of Belgrade, Jove Ilića 154, 11000 Belgrade, Serbia
marija.boricic@fon.bg.ac.rs

Filippo Calderoni

The bi-embeddability relation for countable abelian groups

joint work with Simon Thomas

We analyze the Borel complexity of the bi-embeddability relation for different classes of countable abelian groups. Most notably, we use the Ulm theory to prove that bi-embeddability is incomparable with isomorphism in the case of torsion groups, and p-groups for any fixed prime number p. As I will explain, our result contrasts the arguable thesis that bi-embeddability for countable abelian p-groups has strictly simpler complete invariants than isomorphism.

Dipartimento di Matematica G. Peano, Università di Torino
filippo.calderoni@unito.it

Davide Catta

Natural language variants of universal quantification in first order modal logic

joint work with Alda Mari, Michel Parigot, and Christian Retoré

Natural languages offer a variety of universal quantifiers, both intra and cross-linguistically. French has three wordings of universal quantification: *chaque* (singular; $\sim$ *each*), *tout* (singular; $\not\sim$ *every*) — and a third one, *tous les* (plural; $\sim$ *all*), left out from this study because it involves second order objects.

Based on linguistic data, and granted parametrization of the lexical functions to both sets of individuals and worlds, we show that *chaque* and *tout* differ in the modal properties of their domains of quantification: with *chaque* the world is fixed (as the actual one), with *tout* the set of worlds varies with the individuals. We treat exceptions as pertaining to individuals for *chaque* and to worlds for *tout*, by forcing conditions on accessibility paths.

We will finally discuss the semantics of *tout* in comparison to the silent quantifier GEN that linguists have described as a generic unselective quantifier binding both individuals and worlds variables, with worlds restricted to 'normal' ones.[1]

References

[1] Claire Beyssade, Alda Mari, and Fabio Del Prete, editors. ***Genericity***, Oxford University Press, 2013.

[2] Melvin Fitting and Richard L. Mendelsohn, ***First Order Modal Logic***, Springer, 2007.

[3] Alda Mari, and Christian Retoré, *Conditions d'assertion de* chaque *et de* tout *et règles de déduction du quantificateur universel*, ***Travaux de linguistique***, vol. 72 (2016), pp. 89–106.

LIRMM, Univ. Montpellier, CNRS, 161 rue Ada, 34090 Montpellier, France
davide.catta@lirmm.fr

Filippo Cavallari

Descriptive Set Theory and Automata

In this talk I introduce a quite recent field of research, that is, the connection between Descriptive Set Theory and Automata Theory. The overlap between these two areas is that in Automata Theory the space which we work with is, in fact, the Cantor space 2^ω, that is a well-known uncountable Polish space. In Descriptive Set Theory subsets of the Cantor space can be stratified through topological hierarchies, like the Borel hierarchy and Wadge hierarchy, while in Automata Theory these spaces can also be studied in terms of "regularity", that is the property of being recognised by an automaton. This double point of view leads to many interesting questions about the interplay between topological complexity and regularity. While we have a complete picture of these relationships in the case of automata on words, the case of automata on trees is still a *terra incognita*. In this talk I will present the state of art of the research on automata on trees and I will show some new results that are concerned with general regular tree languages that lie in low levels of the Borel hierarchy and the Wadge hierarchy.

University of Turin - University of Lausanne
filcavallari88@gmail.com

Yong Cheng

A new characterization of supercompactness and Woodin's Local Universality Theorem

I give a new characterization of supercompactness and use it to prove: if κ is supercompact and the **HOD** Hypothesis holds, then in V_κ, there is a proper class of regular cardinals which are measurable in **HOD**. As a corollary, we have Woodin's Local Universality Theorem.

Even if large cardinals from **V** can become disordered in **HOD** via forcing, this work shows that under the **HOD** Hypothesis and supercompact cardinals, large cardinals in **V** become more regular in **HOD** and are reflected to be large cardinals in **HOD** in a local way.

School of Philosophy, Wuhan University, Wuhan 430072 Hubei, Peoples Republic of China
world-cyr@hotmail.com

Cezary Cieśliński

Axiomatic theories of truth based on Weak and Strong Kleene logic

One of the main research topics in the area of axiomatic theories of truth has been that of assessing their strength. A subtle measure of strength has been proposed by Fujimoto in [2]. Namely, denoting by L_T the result of adding a new predicate '$T(x)$' to the language of arithmetic, we say that the truth theory Th_1 is relatively truth-definable in Th_2 iff there is a formula $\theta(x) \in L_T$ such that for every $\psi \in L_T$, if $Th_1 \vdash \psi$, then $Th_2 \vdash \psi(\theta(x)/T(x))$.

If Th_2 defines the truth predicate of Th_1, then Th_2 is not *conceptually* weaker than Th_1, as Th_2 contains the resources permitting to reproduce the concept of truth of Th_1.

We will compare the conceptual strength of two axiomatic theories of truth: KF and WKF. The first one has been designed to capture Kripke's fixed-point construction based on Strong Kleene logic. The second one is based on the Weak Kleene evaluation schema.

In [2] Fujimoto proved that WKF is relatively truth-definable in KF. However, it has been an open question whether KF is relatively truth-definable in WKF.

We will provide the negative answer to this question, one that does not depend on the choice of language and coding. We consider this remarkable, because various important properties of Weak Kleene fixed-point construction are not absolute in this sense (see [1]).

References

[1] Cain, James and Damnjanovic, Zlatan, *On the Weak Kleene scheme in Kripke's theory of truth*, ***The Journal of Symbolic Logic***, vol. 56 (1991), no. 4, pp. 1452–1468.

[2] Fujimoto, Kentaro, *Relative truth definability of axiomatic truth theories*, ***Bulletin of Symbolic Logic***, vol. 16 (2010), no. 3, pp. 305–344.

Institute of Philosophy, University of Warsaw, Poland
c.cieslinski@uw.edu.pl

Nunzia Cosmo

Notes on the Semantic Refractions of the Intuitionistic Logic

In this paper I develop the evaluation of a possible semantic for the dual-intuitionistic logic. In particular, the purpose of this research is to validate the universalisation of the Kripke's semantics for the modal and intuitionistic logic and also for the dual-intuitionistic logic; the possibility of the exstention of the kripkeian's logic is investigated through the examination of semantic of the Da Costa's paraconsistent logic [2,3], which in Priest it's a fragment of Rauszer's Brouwer-Heyting logic [7]. This research work is conducted by the history of semantic for the logical truths and for the logical laws of intuitionistic logic, from Brouwer theory [1] and from Heyting formalism [4] – with their theoretical developments (G. Kreisel, J. Myhill, A. S. Troelstra) – to Kripke's semantic [6] and to Rauszer and Hiroshis' theories [8,9;5].

References

[1]L. E. J. BROUWER, *Over de grondslagen der Wiskunde*, Amsterdam, 1907.

[2]N. C. A. DA COSTA, *Sistemas Formais Inconsistentes*, Curitiba, Brazil, 1963.

[3]-, *Ensaio sobre os Fundamentos da Logica*, Hucitec, Saõ Paulo, 1980.

[4]A.HEYTING, *Die formalen Regeln der intuitionistischen Logik, Sitzungsberichte der Preussischen Akademie der Wissenschaften, Phys. – Math. Klasse*, 1930, pp. 42-56.

[5]A. HIROSHI, *LK, LJ, Dual Intuitionistic Logic and Quantum Logic*, **Notre Dame J. Formal Logic**, vol. 45 (2004), no. 4, pp. 193-213.

[6]KRIPKE, *A Completeness Theorem in Modal Logic*, **J. Symbolic Logic**, vol. 24 (1959), no. 1, pp. 1-14.

[7]G. PRIEST, R. ROUTLEY, J. NORMAN (eds), *Paraconsistent Logic*, München, 1989.

[8]C. RAUSZER, *Semi-Boolean algebras and thier applications to intuitionistic logic with dual operations*, **Fundamenta Mathematicae**, vol. 83 (1974), no. 3, pp. 219-249.

[9]-, *Applications of Kripke Models to Heyting-Brouwer Logic*, **Studia Logica: An International Journal for Symbolic Logic**, vol. 36 (1977), no.1/2, pp. 61-71.

FEDERICO II
nunzia.cosmo@libero.it

Serena Delli

Combinatorial proofs and Genus of proofs

My goal is to produce a geometrical notion of complexity which to compares graph-theoretical proofs.

Traditionally, when we think of a formula we think of a string of symbols. Nevertheless, it is possible to represent formulae as graphs. This technique is being widely used in computer science in order to translate satisfiability into a geometrical property. However, it is possible to use this approach to characterise validity by expressing proofs in terms of graphs and homomorphisms. This approach gets rid of the logical language. It, thereby, builds syntax-free calculi. An example of this is combinatorial proof[2]. CP is a mathematical formulation of first-order classical logic, where proofs are graph-theoretical and combinatorial, instead of syntactic. Here classical step-by-step proofs are replaced by a homomorphism between cographs. This models a projection of the axiomatic structure into the provable formula.

Due to the move to graph theoretic representations of proof we can no longer use tradition measures to compare the complexity of proofs. I will fill this lacuna by offering a way to measure how difficult is to prove a certain formula or to find a CP. Usually, complexity is measured by the number of steps or symbols used in a derivation. But these methods do not work for graph-theoretical calculi. I propose to take the notion of genus introduced in[1] and to apply it to a modified notion of CP presented in[1] and to apply it to a modified notion of CP presented in[3]. Genus is a completely topological and geometrical notion, thus it is a perfect way to measure complexity of CP.

References

[1] Carbone, A. (2009). Logical structures and genus of proofs. *Annals of Pure and Applied Logic*, 161(2), 139-149.

[2] Hughes, D. J. (2006). Proofs without syntax. *Annals of Mathematics*, 1065-1076.

[3] Straßburger, L. (2017, September). Combinatorial Flows and their Normalisation. In *FSCD 2017* (Vol. 84, pp. 311-3117).

New University of Lisbon
s.delli@campus.fct.unl.pt

Stamatis Dimopoulos

Strong compactness and the continuum function

Strong compactness may be one of the established large cardinal notions in set theory, but it is not robust when it comes to forcing. In particular, it is still open whether it is possible to control the continuum function and at the same time preserve strong compactness, without relying on stronger properties such as supercompactness. In an ongoing work with A. Apter, we look at special cases where the preservation of strong compactness is possible, while having some control over the continuum function.

Initially, we show that assuming only a partial degree of supercompactness, it is possible to violate GCH at a non-supercompact strongly compact cardinal, while preserving the full extent of its strong compactness. Also, we show that certain Easton functions can be realised while preserving the strong compactness of the least measurable limit of supercompact cardinals. Finally, we show how to force a violation of GCH at all strongly compact cardinals, in models where strong compactness coincides with supercompactness.

University of Bristol
`stamatis.dimopoulos@bristol.ac.uk`

Marina Dorzhieva

Computable numberings of infinite countable sets

$\nu(n)$ is a computable numbering of a given c.e. sets family S if $\{\langle x, n\rangle | \ x \in \nu(n)\}$ is a c.e. set. We define computable numberings in analytical hierarchy following to Goncharov-Sorbi approach [6]: $\nu(n)$ is a Π^1_k (Σ^1_k)-numbering of Π^1_k (Σ^1_k)-sets family S if $\{\langle x, n\rangle | \ x \in \nu(n)\}$ is Π^1_k (Σ^1_k)-set.

In attempt to prove that there is no Π^1_k-computable Friedberg numbering of all infinite Π^1_k-sets curious results have been obtained:

- There is no computable numbering of all infinite c.e. sets.
- There is no Π^1_1-computable numbering of all infinite Π^1_1-sets.
- There is no Σ^1_2-computable numbering of all infinite Σ^1_2-sets.
- ZF cannot prove that there is a Σ^1_k-computable numbering of all infinite Σ^1_k-sets for $k > 2$.

References

[1] M.V. Dorzhieva, *Metarecursion elimination from Owings theorem* [in Russian], Vestnik NSU, Mathematics, mechanics, informatics, 2014, Volume14, Number 1, pp. 35-43.

[2] M.V. Dorzhieva, *Friedberg numbering of the family of all Σ^1_2-sets* [in Russian], to appear in Vestnik NSU, Mathematics, mechanics, informatics.

[3] James C. OWINGS, JR, *The meta-r.e. sets, but not the Π^1_1 sets, can be enumerated without repetition*, The Journal of Symbolic Logic, Volume 35, Number 2, June 1970.

[4] R.M. Friedberg, *Three theorems on recursive enumeration*, The Journal of Symbolic Logic, Volume 23, Number 3, June 1958, pp. 309-316.

[5] Ershov Yu. L., *Numbering Theory* [in Russian], Nauka, Moscow (1977).

[6] Goncharov S. S., Sorbi A., *Generalized Computable Numerations and Nontrivial Rogers Semilattices* [in Russian], *Algebra Logika*, **36**, Number. 6, 621–641 (1997); English transl.: *Algebra Logic*, **36**, No. 6, 359–369 (1997).

[7] J. W. Addison, *Some consequences of the axiom of constructibility*, Fundamenta Mathematicae 46.3 (1959): 337-357.

Novosibirsk State University, 2 Pirogova St., Novosibirsk, Russia
dm-3004@inbox.ru

Benedict Eastaugh

Dedekind's equivalences and the foundations of analysis

In *Stetigkeit und irrationale Zahlen* [Dedekind 1872], Dedekind demonstrates the equivalence of central theorems of analysis with the *principle of continuity*, a fundamental completeness axiom stating that every division ('cut') of the rational numbers into an upper and lower half determines a unique real number. §VII of [Dedekind 1872] proves that the monotone convergence theorem and the Cauchy convergence theorem are both equivalent to the principle of continuity. This demonstration that fundamental theorems of analysis are not merely implied by the principle of continuity, but also imply it, was a precursor to later work in reverse mathematics.

[Friedman 1975] proved that formalizations in second order arithmetic of the monotone and Cauchy convergence theorems can be proved equivalent to one another, to a sequential version of the least upper bound axiom, and to the arithmetical comprehension scheme, all within the weak base theory RCA_0. In the terminology of reverse mathematics, the least upper bound axiom, the monotone convergence theorem, and the Cauchy convergence theorem all reverse to ACA_0.

It would be natural to consider Friedman's results as formalizations of Dedekind's equivalences. However, Dedekind's work took place within an informal set theory of much greater expressive power than second order arithmetic. This paper will examine whether Dedekind's equivalence proofs can be more faithfully formalized within a higher-order reverse mathematical framework such as that of [Kohlenbach 2002]. The resulting formalization will then be compared to the now-standard formalization of these theorems in second order arithmetic, both in terms of their proof-theoretic characteristics, and their faithfulness to Dedekind's original presentation.

References

[Dedekind 1872]R. Dedekind. *Stetigkeit und irrationale Zahlen*. Vieweg, 1872.

[Friedman 1975]H. Friedman. Some Systems of Second Order Arithmetic and Their Use. In *Proceedings of the 17th International Congress of Mathematicians, Vancouver 1974*, volume 1, pages 235–242, 1975.

[Kohlenbach 2002]U. Kohlenbach. Foundational and mathematical uses of higher types. In W. Sieg, R. Sommer, and C. Talcott, editors, *Reflections on the Foundations of Mathematics: Essays in Honor of Solomon Feferman*, volume 15 of *Lecture Notes in Logic*, pages 92–116. A. K. Peters, 2002.

Munich Center for Mathematical Philosophy, LMU Munich
benedict@eastaugh.net

Mirko Engler

Pathological well-orderings and proof-theoretic ordinals

We consider proof-theoretic-ordinals for theories of 1st and of 2nd order languages. The former may be defined as the supremum of order types of p.r. well-orderings for which a theory T proves the corresponding schema of transfinite induction. The latter (also called Π^1_1-ordinal) may be defined as as the supremum of order types of p.r. well-orderings whose well-foundedness is provable in T. While it is known [2], that for any $\alpha < \omega_1^{CK}$ there are order-relations $\prec$ (not defining a well-ordering) of order type α s.t. $PA \vdash TI(\prec)$, the Π^1_1-ordinal is often claimed to provide a stable and convenient measure of proof-theoretic strength of theories. Moreover, it is claimed to be not dependent on any special concepts of natural well-orderings.
In this contribution, we extend a result from [1] and first show that

$PA \vdash TI(\prec)$ for any $\alpha < \omega_1^{CK}$, where $\prec$ is of order type α and actually represents a well-ordering. Furthermore, we adapt the construction to prove a similiar result for 2nd order and discuss the resulting problems for defining Π^1_1-ordinals.

References

[1] L. Beklemishev, *Another pathological well-ordering*, ***Logic Colloquium '98, Proceedings of the annual European summer meeting of the Association for Symbolic Logic*** (Prague, Czech Republic), (Samuel R. Buss, Petr Hajek and Pavel Pudlak, editors), Lecture Notes in Logic vol. 13, A K Peters, 2000, pp. 105–108.

[2] G. Kreisel, *A variant to Hilbert's theory of the foundations of arithmetic* , ***The British Journal for the Philosophy of Science***, vol. 4 (1953), no. 14, pp. 107–129.

Department of Philosophy, Humboldt-University Berlin, Unter den Linden 6, 10099 Berlin, Germany
englermi@cms.hu-berlin.de

Monroe Eskew

Generic large cardinals as axioms

Matt Foreman [1] proposed augmenting ZFC with generic large cardinal axioms, which are able to settle many classical independent questions, including CH. Foreman argued that these principles have equal claim to axiomhood as conventional large cardinals. We present some theorems that raise technical problems for this proposal, and then critically examine the claim that these axioms are on par with conventional large cardinals. Finally, we propose incorporating these axioms into a pluralist framework for set theory, which avoids the technical problems but sacrifices the goal of settling problems like CH.

References

[1] Matthew Foreman, *Has the continuum hypothesis been settled?*, ***Lecture Notes in Logic***, vol. 24, pp. 56–75.

KGRC, Vienna
monroe.eskew@univie.ac.at

David Fernández Duque

Logic and topology: some connections

joint work with Christian Retoré

One of the oldest connections between logic and topology is Mc Kinsey and Tarski's seminal result stating that S4 is sound and complete for its topological interpretation over the real line and other topological spaces, a result that has since been extended and sharpened and continues to be studied to this day.

This basic construction naturally yields semantics for intuitionistic logic via the Gödel-Tarski translation, which in turn gives rise to models of first order intuitionistic logics. The (pre)sheaves over a topological space (or a pretopology) yield models for which the intuitioinistic predicate calculus is complete. They can be seen as Kripke models with additional properties and enjoy a structure that is more familiar in common mathematics.

These connections between logic and topology have led to a wealth of applications that have become increasingly relevant with developments in computing. Topological semantics for modal logic make them an efficient tool for spatial reasoning, and topological semantics of intuitionistic logic have led to models of computation including denotational semantics of lambda calculi and more recently the development of Homotopy Type Theory.

In this talk we will give brief overview of these connections between logic and topology. We will discuss both historical and technical aspects of the field.

Department of Mathematics, Ghent University, Krijgslaan 281, B 9000 Gent, Belgium
David.FernandezDuque@UGent.be

Marta Fiori Carones

The strength of a theorem about subgraphs with nice properties

joint work with Paul Shafer and Giovanni Soldà

In 1979 Ivan Rival and Bill Sands [1] proved that each infinite graph G has an infinite subgraph H such that each vertex of G is adjacent to none or to one or to infinitely many vertices of H. This statement, showing the existence of a substructure with some property in every infinite graphs, resembles Ramsey's theorem, which assures that each infinite graph has a complete or a totally disconnected subgraph. Actually, the authors presented the statement as a variation of Ramsey's theorem because, while renouncing to the complete information about H itself, it gives some information about the adjacency structure of H with respect to G. Despite the superficial similarity with Ramsey's theorem, this principle is not what in reverse mathematics is called a 'Ramsey's type principle', in fact only if the graph is locally finite each subgraph of a solution is still a solution.

We investigated this statement (restricted to countable graphs) from the viewpoint of reverse mathematics, establishing that it is equivalent to ACA_0. Hence, the coding power of this statement is at least $0'$, but we suspect that it is higher. Moreover, if there is a computable bound to the degree of the vertices of G, then the theorem is computably true.

We will also discuss the reverse mathematics of a related theorem of Rival and Sands proved in the same paper: each countably infinite partial order P of finite width has an infinite chain C such that every element of P is comparable with either none or cofinitely many elements of C.

References

[1] Rival, Ivan and Sands, Bill, *On the Adjacency of Vertices to the Vertices of an Infinite Subgraph*, ***Journal of the London Mathematical Society***, vol. 2 (1980), no. 3, pp. 393–400.

Dipartimento di Scienze Matematiche, Informatiche e Fisiche, Università degli Studi di Udine, via delle Scienze 206, 33100 Udine, Italia
fioricarones.marta@spes.uniud.it

Michael P. Fourman

A semantics for Brouwer's creating subject

Brouwer's second act of intuitionism introduced choice sequences and spreads. His key induction theorems, the Fan theorem and Bar Induction, are justified by reflection on the activities of a creating subject.

Beth models and Kreisel's theory of the creative subject both model a creating subject whose constructions depend on increasing finite information about some choice sequence α. Neither of these approaches provides an account of the freedom of the creating subject to introduce a new choice sequence, β, independent of all past or future constructions based on α.

We have shown [1] that the induction principles are constructively valid in certain topos models. These models are constructed, in the impredicative constructive logic of topoi, as sheaves over topological sites — particular categories of formal spaces with an open cover topology. Monoid models, such as the monoid of continuous functions on formal spaces [2] provide particularly simple examples of topological sites. Formal topology [3] provides a predicative counterpart to the impredicative (logic of topoi) development of formal spaces.

We develop predicative counterparts of the monoid models given by open endomorphisms of the formal Baire and Cantor spaces, each with the open cover topology.

We argue that, just as a Beth model provides a formal account of the intuitionist interpretation of logical connectives, in terms of the activities of the creating subject, so these monoid models and their predicative counterparts provide formal accounts of the introduction of a new choice sequence.

We interpret a proof of the validity of the induction theorems in these models, as a formal model of an informal intuitionistic reflection on the activities of the creating subject, that provides a Brouwerian proof of these principles.

References

[1] Michael P. Fourman, *Notions of Choice Sequenc*, ***The L.E.J. Brouwer centenary symposium*** (Noordwijkerhout), (D. van Dalen and A. S. Troelstra, editors), North-Holland Pub. Co., 1981, pp. 91–105.

[2] Chuangjie Xu and Martín Escardó , *A Constructive Model of Uniform Continuity*, ***TLCA 2013: Typed Lambda Calculi and Applications*** (Eindhoven), (Masahito Hasegawa, editor), vol. 7941, Springer, 2013, pp. 236–249.

[3] Steven Vickers, *Compactness in locales and in formal topology*, ***Annals of Pure and Applied Logic***, vol. 137 (2006), no. 1–3, pp. 413–438.

Laboratory for Foundations of Computer Science, The University of Edinburgh,
10 Crichton Street, Edinburgh EH8 9AB, Scotland, UK
michael.fourman@ed.ac.uk

Emanuele Frittaion

The strange case of Goodman's conservation result

Goodman's theorem (after Nicolas D. Goodman) asserts that adding the axiom of choice to intuitionistic arithmetic in all finite types HA^ω yields a system which is conservative over Heyting arithmetic HA. This is in sharp contrast with classical arithmetic in all finite types. In fact, the combination of choice with classical logic results in a system as strong as full second-order arithmetic.

There are several proofs of this result. Probably too many. Arguably, the most direct and complete proof is in Goodman [1] and is based on a realizability interpretation which combines Kleene recursive realizability with Kripke semantics. I will spotlight the main ideas and present a modified version of Goodman realizability that allows to show that intuitionistic arithmetic in all finite types augmented with both choice and extensionality is also conservative over Heyting arithmetic [2]. The first proof of the extensional case was given by Michael Beeson [3]. Other proofs can be found in Gordeev [4] and, more recently, in van den Berg and Slotte [5].

References

[1] Nicolas Goodman, *Relativized realizability in intuitionistic arithmetic of all finite types*, ***J. Symbolic Logic***, vol. 43(1), pp. 23–44, 1978.
[2] Emanuele Frittaion, *On Goodman realizability*, Accepted for publication in ***NDJFL***, 2018.
[3] Michael Beeson, *GoodmanâĂŹs theorem and beyond*, ***Pacific J. Math.***, 84(1):1–16, 1979.
[4] Lev Gordeev, *Proof-theoretical analysis: weak systems of functions and classes*, ***Ann. Pure Appl. Logic***, 38, pp. 1–121, 1988.
[5] Benno van den Berg and Lotte van Slooten, *Arithmetical conservation results*, ***Indagationes Mathematicae***, 29(1), 260–275, 2017.

Departamento de Matemática, Universidade de Lisboa, Portugal
emanuelefrittaion@gmail.com

Xiaoxuan Fu

Completeness and decidability of the tense logic of integers periods

Tense logic, which extends the classical truth-functional propositional logic by adding four tense operators, was introduced by Arthur Prior in [1] [2], and is recently developed by philosophers, logicians and computer scientists in a variety of ways. In exploring these variations, a natural issue emerges of whether they have the completeness and the decidability.

In this paper, we concentrate on a fundamental model of integers periods $\mathbb{Z}$-model. Van Benthem offers an axiomatization of it, and furthermore, gives a sketch of its completeness proof in [5]. Compared with his proof, we adopt a more direct approach — the step-by-step method introduced in [3][4] to achieve this goal. This method can be taken as a constructive way to build a model through stepwise selections of the maximal consistent sets from the canonical model. It ensures us to exclude the unwanted features of the canonical model in our constructive model.

Our construction begins with selections of the restricted maximal consistent sets (for the lack of the compactness), and sequentially approximates $\mathfrak{M} = \langle \mathfrak{F}, V \rangle$ such that $\mathfrak{F}$ is isomorphic to the integers by eliminating counter examples at each step. Finally we turn to the decidability. A series of the theorems in the $\mathbb{Z}$-tense logic are given to show that the finite model property fails. Instead, we apply the Rabin Theorem to demonstrate the decidability of the $\mathbb{Z}$-tense logic.

References

[1] Prior, A. N., ***Time and Modality***, Oxford: Clarendon Press. 1957.

[2] Prior, A. N., ***Past, Present and Future.***, Oxford: Clarendon Press. 1967.

[3] Blackburn, P., M. de Rijke and Y. Venema, ***Modal Logic***, Cambridge: Cambridge University Press. 2002. pp.225–230.

[4] Gabbay, D. M. and F. Guenthner (eds.), ***Handbook of Philosophical Logic***, 2nd Edition, Volume 7, Netherlands: Springer. 2010. pp. 1–26.

[5] van Benthem, J., ***The Logic of Time***, Netherlands: Springer. 1983.

Department of Philosophy, Humanity school, Tsinghua University, Beijing, 100084, PRC
xfuuva@gmail.com

Nicola Gigante

One-pass tree-shaped tableau methods for Linear Temporal Logic

Linear Temporal Logic (LTL) is the de-facto standard language used to express properties of computations in the fields of formal verification of hardware and software systems [6] where a system is usually checked against LTL specifications describing safety, liveness, and other interesting properties. In this context, it is important to be able to identify mistakes in the specification of such properties, such as vacuously true formulae, contradicting requirements, and so on. The satisfiability problem, *i.e.* finding whether there exists a model satisfying a given formula, is therefore an important problem in the field, which has attracted much research effort in the last decades.

Tableau methods are among the first techniques proposed to solve the satisfiability problem of LTL formulae. Classic tableau methods for LTL are graph-shaped, as they build a graph structure where a model for the formula can be found, if it exists, among the paths of the graph. Although interesting from a theoretical point of view, these methods are not practical in the majority of cases, because of the high cost of building the huge graph structure. Many authors worked on removing this limitation, by proposing incremental [3] and one-pass systems [8].

The present talk describes a recently proposed one-pass tableau method for LTL [7] which is purely tree-shaped, allowing the different search branches to proceed in a completely independent way. This system, easier to explain and present than earlier solutions, has also proved to be efficiently implemented [2] and easily parallelized [5]. The simple structure of the tableau rules allows it to be easily augmented to support different LTL extensions: it has been extended to support past operators [4], and work is ongoing to extend it to real-time temporal logics [1].

References

[1] R. Alur and T. A. Henzinger, *A Really Temporal Logic*, ***Journal of the ACM***, vol. 41 (1994), no. 1, pp. 181–204.

[2] M. Bertello, N. Gigante, A. Montanari, and M. Reynolds, *Leviathan: A new LTL satisfiability checking tool based on a one-pass tree-shaped tableau*, ***Proc. of the 25th International Joint Conference on Artificial Intelligence***, IJCAI/AAAI Press, 2016, pp. 950–956.

[3] A. R. Cavalli and L. F. del Cerro, *A Decision Method for Linear Temporal Logic*, ***Proc. of the 7th International Conference on Automated Deduction***, Springer, 1984, pp. 113–127.

[4] N. Gigante, A. Montanari, and M. Reynolds, *A one-pass tree-shaped tableau for LTL+Past*, ***Proc. of the 21st International Conference on Logic for Programming, Artificial Intelligence and Reasoning***, vol 46 of *EPiC Series in Computing*, 2017, pp. 456–473.

[5] J. C. McCabe-Dansted and M. Reynolds, *A parallel linear temporal logic tableau*, ***Proc. of the 8th International Symposium on Games, Automata, Logics and Formal Verification***, vol 256 of *EPTCS*, 2017, pp. 166–179.

[6] A. Pnueli, *The Temporal Logic of Programs*, ***Proc. of the 18th Annual Symposium on Foundations of Computer Science***, IEEE Computer Society, 1977, pp. 46–57.

[7] M. Reynolds, *A New Rule for LTL Tableaux*, ***Proc. of the 7th International Symposium on Games, Automata, Logics and Formal Verification***, vol 226 of *EPTCS*, 2016, pp. 287–301.

[8] S. Schwendimann, *A new one-pass tableau calculus for PLTL*, ***Proc. of the 7th International Conference on Automated Reasoning with Analytic Tableaux and Related Methods***, vol 1397 of *LNCS*, 1998, pp. 277–292.

University of Udine, Italy
gigante.nicola@spes.uniud.it

BALTHASAR GRABMAYR

A step towards a coordinate free version of Gödel's Second Theorem

[2] locates three sources of indeterminacy in the formalization of a consistency statement for a theory T: (I) the choice of a proof system, (II) the choice of a coding system and (III) the choice of a specific formula representing the axiom set of T. According to [2], "Feferman's solution [1] to deal with the indeterminacy is to employ a fixed choice for (I) and (II) and to make (III) part of the individuation of the theory" (p. 544). [2]'s own approach rests on fixed choices for (II) and (III) but is independent of (I).

The primary result of this talk is to eliminate the dependency on (II), by proving the invariance of Gödel's Second Theorem with regard to acceptable numberings. This involves two steps. Firstly, I discuss the notion of acceptability of a numbering and argue that the computability of a numbering is a necessary condition for its acceptability. A precise notion of computability then allows the formerly vague invariance claim to be restated as a (meta-)mathematical theorem, whose proof will be outlined in the second part of the talk.

References

[1] S. FEFERMAN, *Arithmetization of metamathematics in a general setting*, ***Fundamenta Mathematicae***, vol. 49 (1960), pp. 35–92.

[2] A. VISSER, *Can we make the second incompleteness theorem coordinate free?*, ***Journal of Logic and Computation***, vol. 21 (2011), no. 4, pp. 543–560.

Institut für Philosophie, Humboldt-Universität zu Berlin, Unter den Linden 6, 10099 Berlin, Germany
balthasar.grabmayr@gmx.net

EDUARDO HERMO REYES

A finitely supported frame for the Turing Schmerl calculus

In [1] we introduced the propositional modal logic **ЦС** (which stands for Turing Schmerl Calculus) which adequately describes the provable interrelations between different kinds of Turing progressions. In [2] we defined a model $\mathcal{J}$ which is proven to be a universal model for **ЦС** based on the intensively studied Ignatiev's universal model for the closed fragment of **GLP** (Gödel Löb's polymodal provability logic).

In this talk we will present a new universal frame $\mathcal{H}$. Ignatiev's model and $\mathcal{J}$ share the same domain built from specials sequences of ordinals. For this new frame, we shall consider only sequences with finite support. We will show the completeness of **ЦС** w.r.t. $\mathcal{H}$ and as a consequence, that every world in the domain is modally definable.

References

[1] Hermo-Reyes, E. and Joosten, J. J., *The logic of Turing progressions*, ArXiv:1604.08705, 2017.

[2] ——— *Relational semantics for the Turing Schmerl calculus*, ArXiv:1709.04715, 2018.

[3] Ignatiev, K., *On strong provability predicates and the associated modal logics*, ***The Journal of Symbolic Logic***, vol. 58 (1993), no. 1, pp. 249–290.

[4] Schmerl, U. R., *A fine structure generated by reflection formulas over primitive recursive arithmetic*, ***Logic Colloquium '78***, (Mons, 1978), Stud. Logic Foundations Math, vol. 97, North-Holland, 1979, pp. 335–350.

Department of Philosophy, University of Barcelona, Carrer de Montalegre 6, 08001 Barcelona Spain
ehermo.reyes@ub.edu

Harold T. Hodes

Modal Logic out of Moodal Logic

Let Fml be the set of formulas generated from a set of propositional variables using the logical constants $\bot$, $\supset$, $\Box$ (and &, $\vee$, $\Diamond$ for the intuitionistic case). Form the set $MFml$ of marked formulas by prefixing a formula with a moodal markers: $\mathbf{0}$ and $\mathbf{1}$ (and $\mathbf{1}^+$ for the intuitionistic case). Note: moodal markers are not operators – they cannot be iterated. Heuristic: $\mathbf{0}$ indicates acceptance as true at the actual world; $\mathbf{1}$ indicates acceptance as true at an arbitrary accessible world; $\mathbf{1}^+$ indicates acceptance as true at an arbitrary accessible$^+$ world. Classical and intuitionistic normal modal logics turn out to be $\mathbf{0}$-level fragments of conceptually deeper moodal logics.

Using marked formulas, we can formalize many normal modal logics so as to characterize $\Box$ (and $\Diamond$ in the intuitionistic case) by introduction and elimination rules: the introduction rules for $\Box$ (and $\Diamond$) "freezes" a formula φ marked by $\mathbf{1}$ (or perhaps $\mathbf{1}^+$ in the intuitionistic case) into $\Box\varphi$ (and $\Diamond\varphi$) marked by $\mathbf{0}$; the corresponding elimination rules "unfreezes" $\mathbf{0}\Box\varphi$ (and $\mathbf{0}\Diamond\varphi$).

We will introduce the modal moodal logics Intuitionistic K [Classical K] by model-theoretically defining IK-consequence [CK-consequence] on marked formulas. Our model-theory for IK builds on Plotkin and Sterling's semantics; for CK it builds on Kripke's semantics.

This talk will focus on Natural Deduction formalizations for these logics. Necessitation and the standard axioms are not proof-theoretically "rock-bottom": necessitation is a derived rule; the familar K-axioms, and in the intuitionistic case the other axioms offered by Plotkin and Sterling, are all provable. Time permitting, I will discuss some other familiar normal modal logics.

Note added May 1 2018: I have figured out how to avoid using $\mathbf{1}^+$ in the intuitionistic case.

Cornell University
hth3@cornell.edu

Furio Honsell

Reversible Computation and Principal Types in $\lambda^!$-calculus

joint work with Alberto Ciaffaglione, Pietro Di Gianantonio, Furio Honsell, Marina Lenisa, and Ivan Scagnetto

In [1], S.Abramsky discusses *reversible computation* in a game-theoretic setting using partial *involutions*, *i.e.* functions such that $f(u) = v \Leftrightarrow f(v) = u$. The construction is a special case of a general categorical paradigm [3, 4], which amounts to defining a *combinatory algebra* starting from a *Geometry of Interaction* (GoI) *Situation* in a traced symmetric monoidal category. Involutions amount to history-free strategies and apply according to *GoI symmetric feedback*/Girard's *Execution Formula*.

We highlight a *duality* between the GoI interpretation of a λ-term as an involution and its *principal type* w.r.t. an *intersection types discipline* for a refinement of λ-calculus inspired by Linear Logic, the $\lambda^!$-calculus.
The grammar of types is: $\mu ::= \alpha \mid \mu \to \mu \mid !\mu \mid ¡\mu \mid \mu \wedge \mu$.
The grammar of $\lambda^!$-terms is: $M ::= x \mid MN \mid \lambda x.M \mid \lambda !x.M \mid !M$, where λ-abstractions can be taken only if x occurs at most once and is

not in the scope of a !. Reduction rules are extended with a *!-pattern* β-reduction.

We define inductively the judgements: "$\Vdash M : \sigma$", "the term M has principal type scheme σ", and "$\mathcal{T}(\alpha, \sigma) = u \leftrightarrow v$", "the type-variable α in the principal type σ generates the component $u \leftrightarrow v$ of an involution". We have:

THEOREM. *Given $M, N \in \Lambda^!$ such that $\Vdash M : \sigma_1 \to \sigma_2$, $\Vdash N : \tau$,*
$\cdot\ f_N = \{u \leftrightarrow v \mid \exists \alpha \in \tau.\ \mathcal{T}(\alpha, \tau) = u \leftrightarrow v\}$
$\cdot\ f_M \bullet_{GoI} f_N = \{u \leftrightarrow v \mid S = MGU(\sigma_1, \tau) \wedge \exists \alpha \in S(\sigma_2).(\mathcal{T}(\alpha, S(\sigma_2)) = u \leftrightarrow v)\}$,
where f_N denotes the interpretation of N in GoI, $\bullet_{GoI}$ denotes application in GoI, and MGU denotes the "most general unifier".

The above theorem unveils three conceptually independent, but ultimately equivalent, accounts of *application* in the λ-calculus: *β-reduction*, GoI application of involutions, and *unification* of principal types. Furthermore, we prove that involutions are denotations of combinators iff they generate the principal type of a λ-term, thus answering an open question raised in [1].

The present work extends [2], where the purely affine fragment of the GoI combinatory algebra of involutions and purely affine λ-calculus have been investigated.

References

[1] S. Abramsky, *A Structural Approach to Reversible Computation*, ***Theoretical Computer Science***, vol. 347 (2005), no. 3 pp. 441-464.

[2] A. Ciaffaglione, F. Honsell, M. Lenisa, I. Scagnetto, *Linear λ-calculus and Reversible Automatic Combinators*, submitted, Apr. 2018.

[3] S. Abramsky, E. Haghverdi, P. Scott, *Geometry of Interaction and linear combinatory algebras*, ***Mathematical Structures in Computer Science***, vol. 12 (2002), no. 5, pp. 625–665.

[4] S. Abramsky, M. Lenisa, *Linear realizability and full completeness for typed lambda-calculi*, ***Annals Pure Applied Logic***, vol 134 (2005), nos. 2-3, pp. 122-168.

DMIF, Udine University - UDINE - ITALY
furio.honsell@gmail.com

Radek Honzik

Compactness principles and cardinal invariants

joint work with Šárka Stejskalová

We say that a regular cardinal κ has the *tree property* if every κ-tree has a cofinal branch. If κ is equal to μ^+ for some μ, we say that κ satisfies the *stationary reflection* if for every stationary subset S of $\kappa \cap \mathrm{cof}(<\mu)$ there is a limit ordinal $\alpha < \kappa$ with uncountable cofinality such that $S \cap \alpha$ is stationary (if κ is regular and limit, we may consider any stationary subset S of κ). Note that there are many variants of these compactness principles mentioned in the literature.

We will discuss the effect of compactness principles, such as the tree property or stationary reflection mentioned above, on the values of cardinal invariants (e.g. $\mathfrak{c}$, $\mathfrak{u}$, $\mathfrak{b}$, etc., and their generalizations for uncountable κ, $\mathfrak{c}(\kappa)$, $\mathfrak{u}(\kappa)$, etc.). We will compare the available results and techniques for the Cantor/Baire space at ω, with the generalized spaces 2^κ and κ^κ for an uncountable κ. We will discuss local results (in which we treat κ individually), but also some global variants (in which we study the behaviour of the invariants at more cardinals at once).

Department of Logic, Charles University, Jana Palacha 2, 116 38 Praha 1, Czech Republic
radek.honzik@ff.cuni.cz

Anna Horská

What is the height of Gentzen's reduction trees?

In his consistency proof of 1935 [1], which was not published before 1974, Gentzen constructs reduction procedures for sequents that are derivable in Peano arithmetic, whereas contradictory sequents have no reduction procedures. Reduction procedures generate reduction trees which we interpret as cut-free infinitary derivations. A cut elimination theorem is used to build reduction trees; Gentzen calls it *Hilfssatz* in this particular proof. The Hilfssatz is interesting because of two reasons: (1) The cut elimination strategy applied there eliminates always an uppermost cut, regardless of its complexity. (2) The proof of Hilfssatz makes use of transfinite induction on the height of reduction trees that have been constructed so far. It can be shown that this transfinite induction is the only tool in the consistency proof that cannot be formalized in PA. That is why the height of reduction trees is of the crucial importance.

Based on the analysis of Gentzen's cut elimination strategy in the Hilfssatz, we determined an upper bound for the heights of reduction trees that belong to sequents which are derivable in Peano arithmetic. Namely, the heights of these reduction trees are less than $\Phi_\omega(0)$ where Φ_ω is the ω-th Veblen function. If Gentzen had applied cut elimination strategy that reduces the cut-rank of the derivation, the heights of reduction trees for sequents derivable in Peano arithmetic would be bounded by ε_0.

We would like to introduce the cut elimination strategy of the Hilfssatz and explain why the upper bound on the height of the reduction trees for sequents derivable in Peano arithmetic may be greater than ε_0. Unfortunately, we haven't proved the lower bound yet.

References

[1] Gerhard Gentzen, *Der erste Widerspruchsfreiheitsbeweis für die klassische Zahlentheorie*, ***Archiv für mathematische Logik und Grundlagenforschung***, vol. 16 (1974), no. 3-4, pp. 97–118.

Department of Logic, Charles University, nám. Jana Palacha 2, 116 38 Praha 1, Czech Republic
horska.anna@post.sk

Adi Jarden

Uniqueness triples from the diamond axiom

joint work with Ari Meir Brodsky

Several applications of set-theoretical principles in abstract elementary classes (AECs in short, a branch of model theory) were discovered. In Shelah's book on AECs [2], he invented a version of domination, called uniqueness triples. Most of the results in [2] depend on the existence of uniqueness triples in a fixed cardinality λ. The last chapter of [2] presents a complicated proof of their existence, assuming weak diamond at the two successive cardinals λ^+ and λ^{++} and a constraint on the number of models of cardinality λ^{++}. Here we present a simple proof for the existence of uniqueness triples in λ. We assume $\diamondsuit(\lambda^+)$ in place of weak diamond in two cardinals, and remove the constraint on the number of models of cadinality λ^{++}.

References

[1] Ari Meir Brodsky and Adi Jarden, *Uniqueness triples from the diamond axiom*, ***Preprint, arXiv:1804.10952***, April 2018.

[2] Saharon Shelah, ***Classification Theory for Abstract Elementary Classes 2***, Studies in Logic: Mathematical logic and foundations, College Publications, 2009.

Department of Mathematics, Ariel University, Ariel 4070000, Israel
jardena@ariel.ac.il

Sándor Jenei

Group representation and Hahn-type embedding for a class of involutive residuated chains with an application in Substructural Fuzzy Logic

Hahn's celebrated embedding theorem asserts that linearly ordered Abelian groups embed in the lexicographic product of real groups. Conrad, Harvey, and Holland generalized Hahn's theorem for lattice-ordered Abelian groups [1]. We prove a representation theorem for a class of involutive residuated semigroups, namely for group-like FL_e-chains which possess only finitely many idempotents.

An FL_e-algebra [2] is a structure $\mathbf{X} = (X, \wedge, \vee, \circledast, \rightarrow_{\circledast}, t, f)$ such that $(X, \wedge, \vee)$ is a lattice, $(X, \leq, \circledast, t)$ is a commutative, monoid, and f is an arbitrary constant. One defines $x' = x \rightarrow_{\circledast} f$ and calls $\mathbf{X}$ involutive if $(x')' = x$ holds. Call $\mathbf{X}$ group-like if it is involutive and $t = f$. Since for involutive FL_e-chains $t' = f$ holds, one extremal situation is the integral case (when t is the top element of the universe and hence f is its bottom one) and the other extremal situation is the group-like case (when the two constants coincide). Prominent classes of group-like FL_e-chains are totally-ordered Abelian groups and odd Sugihara chains, the latter constitute an algebraic semantics of a logic at the intersection of relevance logic and fuzzy logic. These two classes are extremal in the sense that lattice-ordered Abelian groups have a single idempotent element, whereas all elements of any odd Sugihara algebra are idempotent.

The representation uses only linearly ordered Abelian groups and a newly introduced construction, called partial lexicographic product. As a corollary we extend Hahn's theorem to this class of semigroups by showing that any such algebra embeds in some partial-lexicographic product of linearly ordered Abelian groups. As an application for this embedding,

we show the finite strong standard completeness of an axiomatic extension of the Involutive Uninorm Logic **IUL** [4] by $\mathbf{t} \Leftrightarrow \mathbf{f}$.
Acknowledgement: Supported by the GINOP 2.3.2-15-2016-00022 grant.

References

[1] P. F. Conrad, J. Harvey, W. C. Holland, *The Hahn embedding theorem for lattice-ordered groups*, ***Transactions of the American Mathematical Society***, vol. 108 (1963), pp. 143–169.

[2] N. Galatos, P. Jipsen, T. Kowalski, H. Ono, ***Residuated Lattices: An Algebraic Glimpse at Substructural Logics***, Studies in Logic and the Foundations of Mathematics, Elsevier, 2007.

[3] H. Hahn, *Über die nichtarchimedischen Grössensysteme*, ***Sitzungsberichte der Akademie der Wissenschaften, Wien, Mathematik-Naturwissenschaften Klasse***, vol. 116 (1907), no. IIa, pp. 601–655.

[4] G. Metcalfe, F. Montagna, *Substructural fuzzy logics*, ***Journal of Symbolic Logic***, vol. 72 (2007), no. 3, pp. 834–864.

Institute of Mathematics and Informatics, University of Pécs, Ifjúság u. 6., Pécs, Hungary
jenei@ttk.pte.hu

Alexander Jones

An axiomatic theory of truth and paradox

In this paper I present a new axiomatic theory of truth, state key theorems about this theory, and discuss its treatment of the semantic paradoxes.

The theory follows the spirit of Tarskian and contextual approaches to truth: truth is treated as a typed notion (sentences are not true absolutely, but true relative to a particular level of the language) and this allows a broadly classical treatment of truth. Where my theory differs to previous theories is that truth is treated as a binary, rather than unary, predicate, and this allows for quantification over the levels of the language. This results in a more expressive theory of truth, that allows internal proofs of some natural statements that must ordinarily be proven outside of the theory. I state some key theorems about this theory, and why it might be viewed as a relatively attractive alternative to other typed theories of truth, before moving on to a discussion of its relation to the semantic paradoxes.

There are a family of Liar-like and truth-teller-like sentences which the theory has to deal with. Some of these are provable, some of their negations are provable, but all are provably untrue. The theory carries with it an internal definition of 'pathological' sentences. This definition allows Liar-like sentences to be provably untrue, without falling into inconsistency. I provide a philosophical interpretation of this definition as sentences which are not 'truth-apt' and consider some other examples of pathological sentences, in particular those which quantify absolutely generally over all levels of the truth predicate. I propose that this is an interesting new axiomatic theory of truth with intriguing formal features that add new depth to contextualist approaches to the semantic paradoxes.

Department of Philosophy, Cotham House, University of Bristol, Bristol, BS6 6JL
alexander.jones@bristol.ac.uk

Birzhan Kalmurzayev

About Rogers semilattices in Ershov hierarchy

joint work with Sergey Ospichev

Study the cardinality of Rogers semilattices of families of sets in different hierarchies is one of the main questions in numbering theory. Here we concentrate our interest on Ershov hierarchy. In work are proven

Theorem. *For any nonzero ordinal notations* $a, b \in O$ *there exist* $A \in \Sigma^{-1}_a$ *and* $B \in \Sigma^{-1}_b$ *such that* $|\mathcal{R}^{-1}_c(\{A, B\})| = 1$, *where* $max\{a, b\} \leq_o c <_o min\{a +_o b, b +_o a\}$.

Theorem. *For any nonzero ordinal notations* $a, b \in O$ *and for all* $A \in \Sigma^{-1}_a$ *and* $B \in \Sigma^{-1}_b$ *Rogers semilattice* $\mathcal{R}^{-1}_c(\{A, B\})$ *is infinite, where* $a +_o b <_o c$ *or* $b +_o a <_o c$.

With these results we also can prove

Theorem. *There are ordinal notations* $a, b \in O$ *of ordinal* ω^2, *such that Rogers semilattices* $\mathcal{R}^{-1}_a$ *and* $\mathcal{R}^{-1}_b$ *are nonisomorphic.*

Second author was supported by RFBR according to the research project No. 17-01-00247.

Al-Farabi Kazakh National University, Almaty, Kazakhstan
birzhan.kalmurzayev@gmail.com

Annika Kanckos

The No-counterexample Interpretation in an Invertible Sequent Calculus

The no-counterexample interpretation (NCI) claims that there is a realiser for the Herbrand normal form of each theorem of a theory obtainable by primitive recursive means. This is true for PA. The NCI was originally proven by G. Kreisel using the ϵ-substitution method. However, there are alternative proofs of this result. An example is the proof in [2] which is attributed to an idea by Kreisel himself. The proof shows that there is a cut-free infinitary derivation in PA of each theorem and that the necessary bounds for a primitive recursive functional can be coded and assigned to each derivable sequent. The reason for using an infinitary calculus is that cut elimination fails in a finitary calculus for PA.

Nevertheless, for a finitary calculus the proof is obtainable in a G3c calculus where the rules are invertible. The process of extending the calculus with axioms and the proof of invertibility are described in [1]. The induction schema can be added to the calculus through adding one instance of the schema in the antecedent of the derivable sequent. Full cut-elimination is thereby not necessary for the proof. Through structural modifications of the derivation based on invertibility of the rules it is then provable that the Herbrand normal form has a realiser. That the structural modifications are primitive recursive is provable through a full Gödel coding, which is possible for the standard finitary derivations.

References

[1] S. Negri and J. von Plato, ***Structural Proof Theory***, Cambridge University Press, 2001.

[2] H. Schwichtenberg, *Proof theory: Some applications of Cut-elimination*, ***Handbook of mathematical logic*** (J. Barwise, editor), North-Holland Publishing Company, Amsterdam, 1977, pp. 867–895.

Philosophy, University of Helsinki, Unioninkatu 40, Finland
annika.siders@helsinki.fi

VLADIMIR KANOVEI

Countable-to-one uniformization at arbitrary projective level

joint work with Vassily Lyubetsky

THEOREM 1 (with Vassily Lyubetsky, ArXived in [3]). *Let $n \geq 2$. In a suitable generic extension of L, the constructible universe, it is true that*

(1) *all boldface $\boldsymbol{\Sigma}^1_n$ sets $P \subseteq \mathbb{R} \times \mathbb{R}$ are boldface $\boldsymbol{\Delta}^1_{n+1}$-uniformizable, but*
(2) *there is a lightface Π^1_n set $Q \subseteq \mathbb{R} \times \mathbb{R}$, whose all vertical cross-sections are Vitali classes, and which cannot be uniformized by a ROD (real-ordinal-definable) set.*

The forcing notion P involved is a subset of the countable support product Q^{ω_1}, where Q consists of all E_0-large trees in $2^{<\omega}$. Earlier results for $n = 2$ appeared in [1, 2]. Uniformization for sets in Σ^1_2 and below is known classically. For instance every boldface Δ^1_1 set (that is, Borel) with countable cross-sections can be uniformized by a Δ^1_1 set, but this is not necessarily true for Δ^1_1 sets with arbitrary cross-sections.

References

[1] VLADIMIR KANOVEI, VASSILY LYUBETSKY, *Counterexamples to countable-section Π^1_2 uniformization and Π^1_3 separation*, ***Annals Pure and Applied Logic***, vol. 167 (2016), 3, pp. 262–283.

[2] ———, *Non-uniformizable sets of second projective level with countable cross-sections in the form of Vitali classes*, ***Russian Mathematics, Izvestiya***, vol. 82 (2018), 1, to appear. DOI: https://doi.org/10.1070/IM8521.

[3] ———, ***Non-uniformizable sets with countable cross-sections on a given level of the projective hierarchy***, ArXiv e-print: 1712.00769, 2017.

IPPI and RUT-MIIT, Moscow, Russia
kanovei@googlemail.com

Artur Khamisyan

On some universal proof system for all versions of many-valued logics

joint work with Anahit Chubaryan

The current research refers to the problem of constructing some universal Gentzen-like proof system for all versions of propositional many-valued logic (MVL) such that for every variant of MVL any propositional proof system can be presented in described form. Some generalization of Kalmar's proof of deducibility for two-valued tautologies in the classical propositional logic gives us a possibility to suggest some simple method for proving the completeness for described systems.

Let E_k be the set $\left\{0, \frac{1}{k-1}, \ldots, \frac{k-2}{k-1}, 1\right\}$. We use the well-known notions of propositional many-valued formula, which defined as usual from propositional variables with values from E_k, (may be also propositional constants), parentheses (,), and logical connectives &, $\vee$, $\supset$, $\neg$, every of which can be defined by different well known modes. For propositional variable $\boldsymbol{p}$ and $\boldsymbol{\delta}= \frac{i}{k-1}$ $(0 \leqslant i \leqslant k-1)$ we define additionally "exponent" functions:

$\boldsymbol{p^{\delta}}$ as $(p \supset \delta)\&(\delta \supset p)$ with Lukasiewicz's implication (1) exponent,

$\boldsymbol{p^{\delta}}$ as p with $(k-1)-i$ cyclically permuting negation (2) exponent,

and introduce the additional notion of formula: for every formulas A and B the expression $\boldsymbol{A^{B}}$ (for both modes) is formula also. For every propositional variable p in k-valued logic $p^0, p^{1/(k-1)}, \ldots, p^{(k-2)/(k-1)}$ and p^1 in sense of both exponent modes are the **literals**. In every MVL either only **1** or every of values $\frac{1}{2} \leqslant \frac{i}{\boldsymbol{k-1}} \leqslant 1$ can be fixed as ***designated values***. A formula φ with variables $p_1, p_2, \ldots p_n$ is called ***k*-tautology** if for every $\tilde{\delta} = (\delta_1, \delta_2, \ldots, \delta_n) \in E_k^n$ assigning δ_j $(1 \leqslant j \leqslant n)$ to each p_j gives the value 1 (or some value $\frac{1}{2} \leqslant \frac{i}{k-1} \leqslant 1$) of φ.

Universal system (US) for all versions of MVL is defined as follow. For sequent we use the denotation $\boldsymbol{\Gamma} \vdash \Delta$, where $\boldsymbol{\Gamma}$ and Δ are finite (may be empty) sequences (or sets) of propositional formulas. For every literal $\boldsymbol{C}$ and for any set of literals $\boldsymbol{\Gamma}$ the axiom sxeme of propositional system **US is** $\boldsymbol{\Gamma, C \rightarrow C}$. For every formulas $\boldsymbol{A}$, $\boldsymbol{B}$, for any set of literals $\boldsymbol{\Gamma}$, for each $\boldsymbol{\sigma_1}$, $\boldsymbol{\sigma_2}$, $\boldsymbol{\sigma}$ from the set E_k and for

$* \in \{\&, \vee, \supset\}$ the logical rules of US are:

$$\vdash * \frac{\Gamma \vdash A^{\sigma_1} \text{ and } \Gamma \vdash B^{\sigma_2}}{\Gamma \vdash (A * B)^{\varphi_*(A,B,\sigma_1,\sigma_2)}}, \quad \vdash \exp \frac{\Gamma \vdash A^{\sigma_1} \text{ and } \Gamma \vdash B^{\sigma_2}}{\Gamma \vdash (A^B)^{\varphi_{\exp}(A,B,\sigma_1,\sigma_2)}},$$

$$\vdash \neg \frac{\Gamma \vdash A^{\sigma}}{\Gamma \vdash (\neg A)^{\varphi_\neg(A,\sigma)}},$$

literals elimination $\vdash \dfrac{\Gamma, p^0 \vdash A, \Gamma, p^{\frac{1}{k-1}} \vdash A, \dots, \Gamma, p^{\frac{k-2}{k-1}} \vdash A, \Gamma, p^1 \vdash A}{\Gamma \vdash A}$,

where many-valued functions $\varphi_*(A,B,\sigma_1,\sigma_2)$, $\varphi_{\exp}(A,B,\sigma_1,\sigma_2)$ and $\varphi_\neg(A,\sigma)$, must be defined individually for each version of MVL such, that

1. formulas $A^{\sigma_1} \supset (B^{\sigma_2} \supset (A * B)^{\varphi_*(A,B,\sigma_1,\sigma_2)})$, $A^{\sigma_1} \supset (B^{\sigma_2} \supset (A^B)^{\varphi_{\exp}(A,B,\sigma_1,\sigma_2)})$ and $A^{\sigma} \supset (\neg A)^{\varphi_\neg(A,\sigma)}$ must be k-tautology in this version,

2. if for some σ_1, σ_2, σ the value of $\sigma_1 * \sigma_2$ ($\sigma_1^{\sigma_2}, \neg\sigma$) is one of ***designed values*** in this version of MVL , then $(\sigma_1 * \sigma_2)^{\varphi_*(\sigma_1,\sigma_2,\sigma_1,\sigma_2)}) = \sigma_1 * \sigma_2((\sigma_1^{\sigma_2})^{\varphi_{\exp}(\sigma_1,\sigma_2,\sigma_1,\sigma_2)} = \sigma_1^{\sigma_2}$, $(\neg\sigma)^{\varphi_\neg(\sigma,\sigma)} = \neg\sigma)$.

Theorem. *Any sequent* $\vdash$ ***A*** *is derived in* **US** *iff formula* ***A*** *is k-tautology.*

Department of Informatics and Applied Mathematics, Yerevan State University, 1 Alex Manoogian, Armenia
Artur.Khamisyan@gmail.com

Anja Komatar

Ramsey classes and partial orders

A class $\mathcal{K}$ of finite structures is a *Ramsey class* if given any two structures $A, B \in \mathcal{K}$ there exists a structure $C \in \mathcal{K}$ such that for any k-colouring of substructures of C isomorphic to A there exists a substructure of C isomorphic to B with all of its substructures isomorphic to A monochromatic. Suppose we are given a class $\mathcal{K}_0$ of finite structures in a language L_0. If $\mathcal{K}_0$ contains structures that are not rigid (structures that have non-trivial automorphisms), $\mathcal{K}_0$ is often easily shown not to be Ramsey. It is thus often necessary to expand the language L_0 to a language $L = L_0 \cup \{\prec\}$ by adding an additional binary relation symbol $\prec$. An *order structure* A for L is a structure A for which $\prec^A$ is a linear ordering. An *order class* $\mathcal{K}$ for L is one for which all $A \in \mathcal{K}$ are order structures. In an order class $\mathcal{K}$ then all structures are rigid. Additionally, the class

$\mathcal{K}$ has the *ordering property* if given any $A_0 \in \mathcal{K}_0$ there exists a $B_0 \in \mathcal{K}_0$ such that for any structures $A = (A_0, \prec^A)$ and $B = (B_0, \prec^B)$ in $\mathcal{K}$ we have that A is a substructure of B. The paper [2] explores the links between structural Ramsey theory and topological dynamics. It shows that results about Ramsey classes and classess with ordering property yield results about extremely amenable groups and universal minimal flows.

Homogeneous partial orders together with expansions by unary predicates, referred to as *shapes* or *colours*, are classified in [1]. We expand the corresponding Fraïssé classes to order classes and seek the Ramsey classes of shaped ordered partial orders with ordering property. We state the corresponding topological dynamics results.

References

[1] S. T. de Sousa and J. K. Truss, *Countable homogeneous coloured partial orders*, ***Dissertationes Mathematicae***, vol. 455 (2008), pp. 1-48.

[2] A.S. Kechris, V.G. Pestov and S. Todorcevic, *Fraïssé limits, Ramsey theory, and topological dynamics of automorphism groups*, ***Geometric and functional analysis***, vol. 15 (2005), no. 1, pp. 106-189.

School of Mathematics, University of Leeds, Woodhouse Lane, Leeds, UK
mmak@leeds.ac.uk

Ruslan Kornev

Computable metrics above the standard real metric

We study the notion of computable categoricity for the space of real numbers viewed as a metric space with the rationals as the canonical dense subset. In other words, our goal is to construct computably inequivalent (in the sense of TTE [1]) computable metrics on the real line. In this setting, the following notions of computable reducibility of metrics are considered. Let (X, τ) be a Polish space with a dense subset W, let ρ and ρ' be complete metrics on X that induce topology τ. We say that ρ is computably reducible to ρ' ($\rho \leq_c \rho'$) if δ_ρ is computably reducible to $\delta_{\rho'}$, where δ_ρ and $\delta_{\rho'}$ are respective Cauchy representations of spaces (X, ρ, W) and (X, ρ', W). Also, we say that ρ is weakly reducible to ρ' ($\rho \leq_{ch} \rho'$) if there is a $(\delta_\rho, \delta_{\rho'})$-computable autohomeomorphism of X.

In [2] it was proved that there exists an infinite sequence of computable metrics on $\mathbb{R}$ which are c-reducible to the standard metric $\rho_\mathbb{R}$ and not ch-reducible to each other, thus, $\mathbb{R}$ has computable dimension ω. In the present talk we construct computable metrics above $\rho_{\mathbb{R}}$ in the ordering $\leq_{ch}$. More precisely, the following result is established.

THEOREM 1. *There exists a sequence of computable metrics* $(\rho_i)_{i\in\omega}$ *on* $\mathbb{R}$ *such that* $\rho_{\mathbb{R}} \leq_c \rho_i$ *for all* $i \in \omega$ *and the ordering* $((\rho_i)_{i\in\omega}, \leq_{ch})$ *is isomorphic to* $\omega^{<\omega}$.

References

[1] K. Weihrauch, ***Computable Analysis***, Springer, 2000.

[2] R. Kornev, *Reducibility of computable metrics on the real line*, ***Algebra and Logic***, vol. 56 (2017), no. 4, pp. 302–317.

Department of Mathematics and Mechanics, Novosibirsk State University, 1 Pirogova St., Novosibirsk 630090, Russia
kornevrus@gmail.com

Jerzy Król

Set-theoretic forcing in low-dimensional differential topology and cosmology

joint work with Torsten Asselmeyer-Maluga, Krzysztof Bielas and Paweł Klimasara

We propose a cosmological model as a mathematical tool for relating smoothness structures on $\mathbb{R}^4$ with the Cohen and random forcings. It refines the model where the initial quantum state defines the lattice of projections $\mathbb{L}(\mathcal{H})$ of a Hilbert space $\mathcal{H}$ [1]. The maximal Boolean subalgebras of $\mathbb{L}(\mathcal{H})$ are typically atomless measure algebras B supporting random forcing. Internal real numbers of the Boolean-valued models V^B parametrize the spacetime points, the smooth regions of the spacetime gain exotic smoothness of $\mathbb{R}^4$. Additionally it predicts the value of the cosmological constant [2].

Cohen forcing distinguishes exotic smoothness structures of $\mathbb{R}^4$. One turns to Calkin algebra $\mathcal{C}(\mathcal{H}) = B(\mathcal{H})/\mathcal{K}$ and $\mathrm{Aut}(\mathbb{L}(\mathcal{H}))$, $\mathrm{Aut}(\mathcal{C}(\mathcal{H}))$. Classically $\mathcal{C}(\mathcal{H})$ is $P(\mathbb{N})/fin$ representing the hyperfinite extensions of Casson handles in nonstandard models of $\mathbb{N}$. The trivial automorphisms of $P(\mathbb{N})/fin$ define operations on handlebodies and diffeomorphisms of exotic R^4's [3]. Additionally if an atlas of $\mathbb{R}^4$ contains transition functions from the Cohen extensions $\mathbf{L}[a]$ of Gödel universe $\mathbf{L}$ it defines exotic R^4.

References

[1] Król, J.; Asselmeyer-Maluga, T.; Bielas, K.; Klimasara, P. From Quantum to Cosmological Regime. The Role of Forcing and Exotic 4-Smoothness. *Universe* 2017, 3, 31.

[2] Asselmeyer-Maluga, T.; Król, J. How to obtain a cosmological constant from small exotic $\mathbb{R}^4$. *Phys. Dark Universe* 2018, 19, p. 66-77.

[3] Król, J. Model and set-theoretic aspects of exotic smoothness structures on $\mathbb{R}^4$, in At the Frontier of Spacetime, ed. T. Asselmeyer-Maluga, Fundamental Theories of Physics vol 183, pp. 217-240, Springer: Switzerland, 2016.

Institute of Physics, University of Silesia, ul. 75 Pułku Piechoty 1, 41-500 Chorzów, Poland
jerzy.krol@us.edu.pl

Sonia L'Innocente

Abelian regularization of rings and modules

joint work with Ivo Herzog

This report aims at describing how Olivier's construction for the commutative regularization of a commutative ring can be generalized to obtain the abelian regularization of an associative ring R. The Ziegler Spectrum of this abelian regularization and its relation with the constructible Cohn spectrum of R, (i.e., the Cohn spectrum equipped with the patch topology) are investigated. It is also shown how Prest's notion of the sheaf of definable scalars on the constructible Cohn spectrum generalizes Wiegand's description of the commutative regularization.

School of Science and Technology, Mathematics Division, University of Camerino, Via Madonna delle Carceri 9, Camerino, Italy
sonia.linnocente@unicam.it

Quentin Lambotte

On Expansions of $(\mathbf{Z}, +, 0)$

joint work with Françoise Point

Call a (strictly increasing) sequence $R = (r_n)$ of natural numbers *regular* if it satisfies the following two conditions:

1. R is eventually periodic modulo m for all $m \geq 2$;

2. $\lim_{n\to\infty} r_{n+1}/r_n = \theta \in \mathbf{R}^{>1} \cup \{\infty\}$ and, if θ is algebraic, then R satisfies a recurrence relation whose characteristic polynomial is the minimal polynomial of θ.

THEOREM 1 ([2]). *Let R be a regular sequence. Then $\mathcal{Z}_R = (\mathbf{Z}, +, 0, R)$ is superstable of U-rank ω.*

The proof follows the strategy used by D. Palacin and R. Sklinos [3] to show that, when $q > 1$, $\mathcal{Z}_{(q^n)}$ and $\mathcal{Z}_{(n!)}$ are superstable, with U-rank ω (the first result was also shown, using different methods, by B. Poizat in [5]). Independently of our work, the results of [3] were generalized, using the same strategy, by Gabriel Conant in [1]. The results in [1] and Theorem 1 have a non trivial overlap: the case $\theta = \infty$ is completely treated by Conant without the periodicity assumption while our result concerning recurrence relations is more general.
A function $\mathtt{A} : \mathbf{N} \to \mathbf{Z}$ of the form $\mathtt{A}(n) = a_0 r_n + a_1 r_{n+1} + \cdots + a_d r_{n+d}$, $a_0, \ldots, a_d \in \mathbf{Z}$, is called an *operator* on R. Let $\mathcal{L}$ be the language

$$\mathcal{L} = \{+, -, 0, 1, D_n | n > 1\} \cup \{R, S, S^{-1}\} \cup \{\Sigma_{\bar{\mathtt{A}}} | \bar{\mathtt{A}} \text{ is a tuple of operators}\},$$

where D_n is a predicate for $n\mathbf{Z}$, S is the successor function on R ($S(r_n) = r_{n+1}$), S^{-1} its inverse and $\Sigma_{\bar{\mathtt{A}}}$ is a predicate for the image of the function $\mathtt{A}_1 + \cdots + \mathtt{A}_k$.

THEOREM 2. *Let R be a regular sequence. Then $Th(\mathcal{Z}_R)$ has quantifier elimination in $\mathcal{L}$.*

Theorem 2 corresponds to known results in Presburger arithmetic (see [6] and [4]), and allows us to prove directly that $\mathcal{Z}_R$ is superstable.

References

[1] GABRIEL CONANT, *Stability and sparsity in sets of natural numbers*, preprint (arXiv 1701.01387)

[2] QUENTIN LAMBOTTE AND FRANÇOISE POINT, *On Expansions of* $(\mathbf{Z}, +, 0)$, preprint (arXiv 1702.04795)

[3] DANIEL PALACIN AND RIZOS SKLINOS, *On Superstable Expansions of Free Abelian Groups*, ***Notre Dame Journal of Formal Logic***, vol. 59 (2018), no. 2, pp. 157–169.

[4] FRANÇOISE POINT, *On decidable extensions of Presburger arithmetic: from A. Bertrand numeration systems to Pisot numbers*, ***Journal of Symbolic Logic***, vol. 65 (2000), no. 3, pp. 1347–1374.

[5] BRUNO POIZAT, *Supergénérix*, ***Journal of Algebra***, vol. 404 (2014), pp. 240–270

[6] A. L. Semenov, *On certain extension of the arithmetic of addition of natural numbers*, ***Mathematics of the USSR-Izvestia***, vol. 22 (1992), no. 2, pp. 401–418.

Département de Mathématique, Université de Mons, 15 Avenue Maistriau, 7000 Mons, Belgique
quentin.lambotte@umons.ac.be

Junguk Lee

Valued hyperfields, truncated discrete valuation rings, and valued fields

In [2], M. Krasner introduced a notion of valued hyperfield analogous to a valued field with a multivalued addition operation, and used it to do a theory of limits of local fields. In [1], P. Deligne did the theory of limits of local fields in a different way by defining a notion of triple, which consists of truncated discrete valuation rings and some additional data. Typical examples of a valued hyperfield and truncated discrete valuation ring are a n-th valued hyper fields, which is quotient of a valued field by a multiplicative subgroup of the form $1 + \mathfrak{m}^n$, where $\mathfrak{m}$ is the maximal ideal of a valuation ring, and a n-th residue ring, which is a quotient of a valuation ring by the n-th power of the maximal ideal.

J. Tolliver in [4] showed that discrete valued hyperfields and triples are essentially same, stated by P. Deligne in [1] without a proof. In [3] W. Lee and the author showed that given complete discrete valued fields of mixed characteristic with perfect residue fields, any homomorphism between the n-th residue rings of the valued fields is lifted to a homomorphism between the valued fields for large enough n. This lifting process is functorial.

Motivated by above results in [4] and [3], we show that given complete discrete valued fields of mixed characteristic with perfect residue fields, any homomorphism between the n-th valued hyperfields of the valued fields can be lifted to a homomorphism between the valued fields for large enough n, which is functorial. We also compute an upper bound of such a minimal n effectively depending only on the ramification index. Most of all, any homomorphism between the first valued hyperfields of valued fields is uniquely lifted to a homomorphism between the valued fields in the case of tamely ramified valued fields. From this lifting result, we prove a relative completeness AKE-theorem via valued hyperfields for finitely ramified valued fields with perfect residue fields.

References

[1] P. Deligne. Les corps locaux de caractéristique p, limits de corps locaux de caractéristique 0. J.-N. Bernstein, P. Deligne, D. Kazhdan, M.-F. Vigneras, Representations des groupes reductifs sur un corps local, Travaux en cours, Hermann, Paris, 119-157, (1984).

[2] M. Krasner. Approximation des corps valués complets de caractéristique $p \neq 0$ par ceux caractéristique 0. 1957 Colloque d'algèbre supérieure, tenu á Bruxelles du 19 au 22 décembre.

[3] J. Lee and W. Lee. On the structure of certain valued fields, preprint.

[4] J. Tolliver. An equivalence between two approaches to limits of local fields, J. of Number Theory, (2016), 473-492.

Institute of Mathematics, Wroclaw University
jlee@math.uni.wroc.pl

Gabriel Lehéricy

Classification of C-minimal groups using quasi-orders

A C-relation is a ternary relation which is naturally interpretable in the set of maximal chains of a tree. The notion of C-minimality was introduced in [3] as a variant of o-minimality, where the order is replaced by a C-relation. The authors of [3] also introduced the notion of C-group, which is a group endowed with a C-relation compatible with the group operation. Two fundamental examples of C-groups are ordered groups and valued groups. We know from [1] that an ordered group is C-minimal if and only if it is o-minimal. Moreover, the authors of [2] gave a complete description of valued C-minimal abelian groups. However, there is still no complete classification of general C-minimal groups. In this talk, we will show that quasi-orders can be used to classify C-minimal groups.

A quasi-order is a reflexive and transitive binary relation. A compatible C-relation on a group canonically induces a total quasi-order, so that quasi-orders can be used to study C-groups. Using quasi-orders, we will show that any C-group is basically a "mix" of ordered and valued groups, in the sense that the group can be decomposed into "fundamental parts" on each of which the C-relation either comes from a valuation or from an order. We will then characterize the C-minimality of the group in terms of these "fundamental parts". In particular, we will show that any abelian C-minimal group is a direct product of o-minimal groups with C-minimal valued groups.

References

[1] F. Delon, *C-minimal structures without the density assumption*, ***Motivic Integration and its Interactions with Model Theory and Non-Archimedean Geometry*** (Raf Cluckers, Johannes Nicaise and Julien Sebag), Cambridge University Press, 2011, pp.51–86.

[2] F. Delon and P. Simonetta, *Abelian C-minimal valued groups*, ***Annals of Pure and Applied Logic***, vol. 168 (2017), pp. 1729–1782.

[3] D. Macpherson and C. Steinhorn, *On variants of o-minimality*, ***Annals of Pure and Applied Logic***, vol. 79 (1996), pp. 165–209.

Université Paris 7-Universität Konstanz
gabriel.lehericy@imj-prg.fr

Johan Lindberg

Some properties of the category of local sets in intuitionistic ramified type theory

In [1] Palmgren introduces a ramified type theory based on intuitionistic logic which can be seen as a modern (constructive) reformulation of Russell and Whitehead's type theory in Principia Mathematica. This theory, called Intuitionistic Ramified Type Theory (IRTT), can be interpreted in Martin-Löf type theory by means of setoids in a natural way, and in this sense clarifies the relation between these two kinds of type theories.

As in [2], from which the terminology is borrowed, one can obtain a category of "local sets" from IRTT as a certain syntactic construction. Based on the formulation of IRTT, being a certain intuitionistic type theory with a ramified power type operation, this can expected to give a natural notion of a predicative elementary topos.

Here we show that this category of local sets in IRTT is a Π-pretopos with a natural number object, and that by extending IRTT with a replacement axiom and an axiom allowing for inductive definitions, the corresponding category defines a ΠW-pretopos.

References

[1] E. Palmgren, *A constructive examination of a Russell-style ramified type theory*, ***The Bulletin of Symbolic Logic***, 24(1), 90-106, 2018.

[2] J.L. Bell, ***Toposes and Local Set Theories: An Introduction***, Clarendon Press, 1988.

Department of Mathematics, Stockholm University, 106 91 Stockholm, Sweden
johan.lindberg@math.su.se

José M. Méndez

Basic quasi-Boolean extensions of relevant logics

joint work with Gemma Robles and Francisco Salto

Let L be a negation relevant logic. A Boolean negation (B-negation) can be introduced in L by adding to it the following axioms: (a1) $(A \wedge \sim A) \rightarrow B$, (a2) $B \rightarrow (A \vee \sim A)$. This way of introducing B-negation in relevant logics suggests the definition of two families of quasi-Boolean negation (QB-negation) extensions of relevant logics. One of them, intuitionistic in character, has a1 but not a2; the other one, dual intuitionistic in nature, has a2 but lacks a1. The aim of this paper is to begin the investigation of both families of QB-negation extensions of relevant logics. B-negation extensions of relevant logics are of both logical and philosophical interest (cf. [1], pp. 376, ff). It is to be expected that QB-negation extensions of the same logics (not considered in the literature, as far as we know) will have a similar logical interest.

References

[1] R. Routley, R. K. Meyer, V. Plumwood, R. T. Brady, ***Relevance logic and their rivals***, vol. 1, Atascadero, CA: Ridgeview Publishing Co., 1982.

Acknowledgements. - Work supported by research project FFI2017-82878-P, financed by the Spanish Ministry of Economy, Industry and Competitiveness.

Universidad de Salamanca. Edificio FES, Campus Unamuno, 37007, Salamanca, Spain
sefus@usal.es

Margarita Marchuk

Decidable $\mathbf{0}'$-categoricity of models which realize only types with low CB ranks

Goncharov[1] investigated computable categoricity restricted to decidable structures. Let $\mathbf{d}$ be a Turing degree. A decidable structure $\mathfrak{M}$ is *decidably* $\mathbf{d}$*-categorical* if for every decidable copy $\mathfrak{N}$ of $\mathfrak{M}$, there exists a $\mathbf{d}$-computable isomorphism from $\mathfrak{M}$ onto $\mathfrak{N}$. There exists a Nurtazin criteria [2] of decidable categoricity. On the base of this criteria Goncharov[1] found sufficient conditions of decidable $\mathbf{d}$-categoricity.

Last summer on the workshop "Aspects of Computation" David Belanger asked if we can obtain sufficient conditions of decidable $\mathbf{0}'$-categoricity which relaxes the requirement from realizing only principal types to realizing only types of Cantor-Bendixson rank ≤ 1. It seemed to be nice and natural hypothesis, but this proved not to be the case.

Theorem 1. *There exists decidable model* $\mathfrak{M}$ *such that:*
(i) the set of complete formulas of $Th(\mathfrak{M})$ *is computable,*
(ii) $\mathfrak{M}$ *is not homogeneous and realize only types of Cantor-Bendixson rank* ≤ 1
(iii) $\mathfrak{M}$ *is not decidably* $\mathbf{0}'$*-categorical.*

Theorem 2. *There exists decidably* $\mathbf{0}'$*-categorical model* $\mathfrak{M}$ *such that:*
(i) the set of complete formulas of $Th(\mathfrak{M})$ *is computable,*
(ii) $\mathfrak{M}$ *is not homogeneous and realize only types of Cantor-Bendixson rank* ≤ 1

References

[1] S.S. Goncharov, *Degrees of autostability relative to strong constructivizations*, ***Proceedings of the Steklov Institute of Mathematics*** , vol. 274 (2011), pp. 105–111.

[2] A.T. Nurtazin, *Strong and weak constructivizations and computable families* , ***Algebra and Logic***, vol. 13 (1974), pp. 177–184.

Sobolev Institute of Mathematics, 4 Acad. Koptyug Ave., Novosibirsk, Russia;
Novosibirsk State University, 2 Pirogova St., Novosibirsk, Russia
margaretmarchuk@gmail.com

José Martínez-Fernández

Four-valued semantics for languages with two sources of pathologicality

Three-valued logics are standardly used to formalize gappy languages, i.e., interpreted languages in which sentences can be true, false or neither. In this talk we would like to explore how to build a four-valued truth-functional semantics for a language which contains two different sources of semantic pathologies that give rise to two different types of gappy sentences. We will concentrate on languages that contain, on the one hand, sentences that lack classical truth value because they do not express a proposition and, on the other, sentences that do express a proposition but, due to some deficiency (be it in the semantics or in the facts of the world) are neither true nor false. The set of semantic values will be 0,1,2,3, with 0 (resp. 1) being assigned to sentences that express a false (resp. true) proposition, 2 to sentences that express a neither true nor false proposition and 3 to sentences that do not express a proposition at all. The main four-valued logic, Belnap-Dunn logic, will be discarded as a possible semantics for these languages, as $2 \wedge 3 = 0$, against the interpretation of the values. The search for a semantics will be conducted among the operators that are monotonic on the order $3 \geq 2$, $2 \geq 0$, $2 \geq 1$. We will determine the best available options and discuss some of their properties. One of them happens to be one of the logics created by M. Fitting [1] as a four-valued generalization of Weak Kleene logic.

References

[1] Melvin Fitting, *Kleene's three-valued logics and their children*, ***Fundamenta Informaticae***, vol. 20 (1994), pp. 113–131.

Logos Research Group, Department of Philosophy, University of Barcelona, Montalegre St. 6, 08001 Barcelona (Spain)
jose.martinez@ub.edu

Samuele Maschio

Two relevant doctrines for constructive topology

joint work with Francesco Ciraulo and Tatsuji Kawai

The Freyd-Grandis completion ([2, 3]) of the category of relations Rel, which exactly coincides with Sambin's category of basic pairs BP in [5], is classically equivalent to the category of suplattices SL; constructively this is not the case. We will show that the construction in [1], corresponding to the category of basic topologies BTop in [5], can be seen as the result of a Grothendieck construction $\int \mathsf{P}$ on a specific doctrine (see [4]) P on SL^{op} obtained by composing the functor $\mathsf{SL}(-,\Omega)$ with $\mathsf{Sub}_{\mathsf{SL}}$ where Ω is the power suplattice $\mathcal{P}(1)$. This construction can be performed in a constructive impredicative framework. At the same time, in such a framework, we show that one can also produce a construction on BP consisting of a doctrine Q which gives rise to a fibration $\int Q \to \mathsf{BP}$ for which, impredicatively, $\int \mathsf{P}$ is equivalent to $\int \mathsf{Q}$. Moreover, the doctrine P can be restricted to locales obtaining the category PTop of positive topologies in [5] as the domain of a fibration over the category Loc.

References

[1] Francesco Ciraulo, and Steven Vickers, *Positivity relations on a locale*, ***Annals of Pure and Applied Logic***, vol. 167 (2016), no. 9, pp. 806–819.

[2] Peter J. Freyd, *Representations in abelian categories*, ***Proceedings of Conference on Categorical Algebra*** (La Jolla), Springer, 1965, pp. 95–120.

[3] Marco Grandis, *Weak subobjects and the epi-monic completion of a category*, ***Journal of Pure and Applied Algebra***, vol. 154 (2000), no. 1-3, pp. 193–212.

[4] Bart Jacobs, ***Categorical Logic and Type Theory***, Studies in Logic and Foundations of Mathematics, Elsevier, 1999.

[5] Giovanni Sambin, ***Positive Topology and the Basic Picture. New structures emerging from Constructive Mathematics***, Oxford Logic Guides, Oxford University press, to appear.

Dipartimento di Matematica, Università di Padova
maschio@math.unipd.it

RUSSELL MILLER

Hilbert's Tenth Problem as a pseudojump operator

joint work with Ken Kramer

When considering subrings of $\mathbb{Q}$, it is natural to define the *HTP-operator* to be the pseudojump operator sending each subset W of the set $\mathbb{P}$ of prime numbers to the set

$$HTP(R_W) = \{f \in \mathbb{Z}[X_1, X_2, \ldots] \ : \ f = 0 \text{ has a solution in } \mathbb{Z}[W^{-1}]\},$$

known as *Hilbert's Tenth Problem* for the subring $R_W = \mathbb{Z}[W^{-1}]$ of $\mathbb{Q}$. We show that every Σ^0_2 Turing degree above $\mathbf{0'}$ is the degree of $HTP(R_W)$ for some Π^0_1 set W. Moreover, this operator does not respect Turing equivalence. Indeed, using the technique of high permitting, we can give an example where the HTP operator reverses the (strict) Turing reductions: $V <_T W$, yet $HTP(R_W) <_T HTP(R_V)$. (The set V here is Π^0_1, while W is Σ^0_1.)

Mathematics Dept., Queens College & CUNY Graduate Center, 65-30 Kissena Blvd. Queens NY 11367, USA
`Russell.Miller@qc.cuny.edu`

RYSZARD MIREK

Linear Perspective in Renaissance

Renaissance mathematicians and geometers like Piero della Francesca and Luca Pacioli refers directly or indirectly to Euclidean geometry. But what is new in the Renaissance concerns linear perspective. Piero della Francesca in Proposition 1.12 shows how to draw in perspective a surface of undefined shape, which is located in profile as a straight line. According to Proposition 1.13, which is known as the first new European theorem in geometry after Fibonacci, one can add a square that represents the object to be drawn in reality in a horizontal plane . Then, we draw from the position of the eye of a hypothetical observer visual rays to the corners of the square. At the same time this proposition can be used to interpret Renaissance paintings of Piero della Francesca's, applying the strict rules of geometry and perspective. The proposition is directly applicable in the masterpieces painted by Piero della Francesca, namely in his The *Flagellation of Christ* and *The Baptism of Christ*. In turn, in Proposition I.8 he shows for the first time that the perspective images of orthogonals converge to a *centric point*. In *De Pictura* Alberti merely assumed the truth of this result.

My goal here is to provide a detailed analysis of the methods of inference that are employed in the Renaissance treatises with particular emphasis on a method of natural deduction that takes into account the importance of diagrams within formal proofs.

Institute of Logic, Pedagogical University of Cracow Podchorazych 2, 30-084 Krakow, Poland
`mirek.r@poczta.fms`

Kenji Miyamoto

The epsilon calculus with equality predicate and Herbrand complexity

joint work with Georg Moser

Hilbert's ε-calculus is based on an extension of the language of predicate logic by a term-forming operator ε_x [1]. Two fundamental results about the ε-calculus, the first and second epsilon theorem, play a role similar to that which the cut-elimination theorem plays in sequent calculus. In particular, Herbrand's Theorem is a consequence of the epsilon theorems. Moser and Zach study the epsilon theorems and the complexity of the elimination procedure underlying their proof, as well as the length of Herbrand disjunctions of existential theorems obtained by this elimination procedure [2]. We extend their results to ε-calculus with equality predicate.

References

[1] D. Hilbert and P. Bernays, ***Grundlagen der Mathematik***, vol. 2, Springer Berlin, 1939.

[2] G. Moser and R. Zach, *The epsilon calculus and Herbrand complexity*, ***Studia Logica***, vol. 82 (2006), no. 1, pp. 133–155.

University of Innsbruck
`Kenji.Miyamoto@uibk.ac.at`

Sanderson Molick

The different ways that logic is said to be revisable

In [3] Hilary Putnam argued that the discovery of some phenomena from quantum mechanics required the revision of classical logic, especially the law of distributivity, for an accurate description. Since then the debate evolved along the problem of knowing whether logic is revisable in face of empirical information. In several papers, such as [2], [1] and [4], it is investigated the problem of the revisable character of logic in connection to logical apriorism, the thesis according to which logical knowledge is a priori and the legitimacy of its laws is not dependent upon empirical information. In general the arguments devised around the debate runs through the following lines: if logic is empirical, then it is not a priori and, therefore, it is revisable. If logic is a priori, then it is not empirical and, therefore, it is unrevisable. From such a standpoint the question of whether logic is revisable is reduced to the question of whether logic is a priori, or empirical. It is certain that these two problems are intimately connected, but it does not follow that they should be conflated. Based on this, the purpose of the present talk is to argue that the problem of the revisability of logic should not be reduced to the problem of the a priori character of logic. Furthermore, we shall argue that logical revisability occurs in different levels of the logical systems, motivated by a plethora of reasons. Logical revisability might accordingly be understood in a number of different ways.

References

[1] Otávio Bueno, *Is logic a priori?*, ***The Harvard Review of Philosophy***, vol. 17 (2010), no. 1, pp. 105–117.

[2] Hartry Field, *The a prioricity of logic*, ***Proceedings of the Aristotelian Society***, vol. 96 (1996), no. 1, pp. 359–379.

[3] Hilary Putnam, *Is logic empirical?*, ***Boston Studies in the Philosophy of Science***, vol. 1 (1969), no. 1, pp. 216–241.

[4] Graham Priest, *Logical disputes and the a priori*, ***Princípios: Revista de Filosofia (UFRN)***, vol. 23 (2016), no. 40, pp. 29–57.

Federal University of Rio Grande do Norte (UFRN)/Ruhr-University of Bochum (RUB)
smolicks@gmail.com

Alberto Molinari

Complexity of HS model checking

A model checking (MC) framework based on the *Halpern and Shoham's modal logic of time intervals* HS, for the verification of *interval properties of systems*, has recently been proposed. HS is the most famous *interval temporal logic*, which features one modality for each of the 13 possible ordering relations between pairs of intervals (the Allen's relations). Here we summarize the main complexity results about MC for HS and its fragments.

MC for (full) HS is known to be decidable with non-elementary complexity [3]. The HS fragment BE, whose modalities can express properties of both interval prefixes and suffixes simultaneously, is $\mathsf{EXPSPACE}$-hard, and this lower bound propagates to full HS [1]. Later, other HS fragments, where properties of interval prefixes and suffixes are expressed separately, were studied, showing that they exhibit lower complexity. MC for $\mathsf{A\overline{A}}$, which only considers properties of future and past intervals (using future and past modalities A and $\mathsf{\overline{A}}$) is in $\mathsf{P^{NP}}$ [2]. The addition of right and left interval extension modalities $\mathsf{\overline{B}}$ and $\mathsf{\overline{E}}$ to $\mathsf{A\overline{A}}$ results in $\mathsf{A\overline{A}\overline{B}\overline{E}}$, whose MC is PSPACE-complete [4]. Analogously, $\mathsf{A\overline{A}B\overline{B}}$ and $\mathsf{A\overline{A}E\overline{E}}$ are PSPACE-complete [1]. Finally, MC for $\mathsf{A\overline{A}B\overline{B}\overline{E}}$ and $\mathsf{A\overline{A}E\overline{B}\overline{E}}$ belongs to $\mathsf{EXPSPACE}$ and is PSPACE-hard [4].

References

[1] Bozzelli L., Molinari A., Montanari A., Peron A., Sala P., *Which Fragments of the Interval Temporal Logic HS are Tractable in Model Checking?*, ***Theoretical Computer Science***.

[2] ——— *Model Checking for Fragments of the Interval Temporal Logic HS at the Low Levels of the Polynomial Time Hierarchy*, ***Information and Computation***.

[3] Molinari, A., Montanari, A., Murano, A., Perelli, G., Peron, A., *Checking interval properties of computations*, ***Acta Informatica***, vol. 53 (6), pp. 587–619.

[4] Molinari A., Montanari A., Peron A., *Model Checking for Fragments of Halpern and Shoham's Interval Temporal Logic Based on Track Representatives*, ***Information and Computation***, vol. 259 (3), pp. 412–443.

DMIF, University of Udine, via delle Scienze 206, Udine, Italy
molinari.alberto@gmail.com

Tommaso Moraschini

Relational semantics, ordered algebras, and quantifiers for deductive systems

joint work with Ramon Jansana

Relational semantics has proved to be a fundamental tool in the mathematical and philosophical understanding of many non-classical logics, including intuitionistic, modal and substructural logics. Nevertheless, the evolution of the *general* theory of relational semantics is far behind that of algebraic semantics. In this talk [4] we present a first abstract approach to relational semantics, in the spirit of Abstract Algebraic Logic [2], which turns out to be connected with the theory of completions of ordered algebras [6], [1], [3].

Even though, for the sake of simplicity, we will confine our discussion to the *local* aspects [5] of relational semantics, our approach can be extended harmlessly to *arbitrary* propositional logics and, in particular, to global consequences of normal modal logics and to arbitrary substructural logics. More in detail, we will consider two basic questions which need to be addressed by any truly general theory of relational semantics:

- Can we make precise the idea that a logic has a local relational semantics?
- In case a logic has a local relational semantics, what are its distinguished relational models?

As a matter of fact, our approach encompasses that of canonical extensions of arbitrary lattices, but diverges from the known approaches when applied to non-lattice based logics. This is due to the fact that our approach is logic-based, and produces completions and relational semantics which reflect the behaviour of the logic under consideration. This makes it especially fruitful in the study of purely intensional fragments (which are not lattice-based). Interestingly enough, as a by-product of our approach, we are able to associate to every local consequence a class of distinguished *ordered algebras*, thus justifying on general grounds the empiric observation that most algebras of logics are intrinsically ordered.

If time allows, we will consider the following problems as well:

- Can we use the distinguished relational semantics of a propositional logic $\vdash$ to introduce a semantically defined first-order extension $\vdash$?
- Can we axiomatize this first-order extension relative to a given axiomatization of the propositional logic $\vdash$?

References

[1] M. J. Dunn, M. Gehrke, and A. Palmigiano. Canonical extensions and relational completeness of some substructural logics. *The Journal of Symbolic Logic*, 70(3):713–740, 2005.

[2] J. M. Font. *Abstract Algebraic Logic - An Introductory Textbook*, volume 60 of *Studies in Logic - Mathematical Logic and Foundations*. College Publications, London, 2016.

[3] M. Gehrke, R. Jansana, and A. Palmigiano. Δ_1-completions of a poset. *Order*, 30(1):39–64, 2013.

[4] R. Jansana and T. Moraschini. Relational semantics, ordered algebras, and quantifiers for deductive systems. Manuscript, 2017.

[5] M. Kracht. *Modal consequence relations*, volume 3, chapter 8 of the Handbook of Modal Logic. Elsevier Science Inc., New York, NY, USA, 2006.

[6] H. M. MacNeille. Partially ordered sets. *Transactions of the Americal Mathematical Society*, 42(3):416–460, 1937.

Institute of Computer Science, Czech Academy of Sciences, Prague, Czech Republic

Joachim Mueller-Theys

The necessity of reforming modern modal logic

Formal rigour seems to be more important to philosophical logic than to mathematics. However, worth is made up by substance, and it is not clear what the content of modern modal logic precisely is.

The subject was conceived to avoid the so-called paradoxes of "material" implication like $\bot \to \phi$ and $\phi \to \top$, but "strict" implication shows them likewise, namely $\bot \Rightarrow \alpha$ $\big($eq. $\Box(\bot \to \alpha)\big)$, $\alpha \Rightarrow \top$.

Possible worlds are already given on the non-modal level. For instance, structures (models) are the possible worlds of predicate logic.

The notion of world seems to have been misleading. Non-equivalent possible worlds are incompatible: the union of their theories is inconsistent. So they cannot exist simultaneously. Isn't the notion of accessibility therefore dubious?

On the basis of a given logic, say PL, it is clear what the logical modalities are: ϕ is *logically necessary* iff $\models \phi$. Accordingly, ϕ is *logically possible* iff the negation of ϕ is not logically necessary, viz. $\not\models \neg\phi$. Thus any atomic proposition p is logically possible—naturally. However, $\Diamond p$ is *not* a theorem of any of the systems of modern modal logic. The deficiency is caused by uniform substitution, which must not be generalised to the modal level. Proper implementation uniquely leads to the semantic system C (like Carnap) and the evident deductive system S of ours. Cf. The Bulletin of Symbolic Logic 20, pp. 238, 264-5.

The logical modalities in the narrow sense can be generalised to the Σ-*modalities:* σ is Σ-*necessary* iff Σ seq σ, σ is Σ-*non-necessary* iff σ is not Σ-necessary, ... , where *seq* $:= \models = \vdash$. We proved that the Σ-modalities exhaust the *objective modalities*, where a necessity in whatever sense is called *objective* if it is closed under seq. (BSL 21, 239-0; BSL 22, 585)

The implementation of our mathematized conception of adequately formalising the Σ-modalities leads to one and only one consequence relation, namely Σ $seq^{\Box}$ α, which can be constructed as well semantically ($\mathcal{M}, V \Vert\!\models_\Sigma \Box\alpha$:iff for all $\mathcal{N} \models \Sigma$ and all W: $\mathcal{N}, W \Vert\!\models_\Sigma \alpha$, $\Sigma \Vert\!\models \alpha$) as deductively ($\Sigma \Vert\!- \alpha$:iff $\Sigma \cup \{\neg\Box\sigma : \Sigma \not\models \sigma\} \vdash_{\mathrm{QNI}} \alpha$).

The transition to this *metalogical extension* forks validity and *general consequence*: $\Vert\!\models \alpha$ does not imply $/\!/\!\!= \alpha$, viz. $\Sigma \Vert\!\models \alpha$ for all Σ, whence a separate deductive characterization of the general consequences is required, which coincide with the theorems of (a suitable quantificational version of) S5 probably. (BSL 23, 211-2)

It's not that laws like (NR), (K), (T), (4), (5) were false: wrong conception, unjustified pluralism, persisting in uniform substitution and monotony, misguided semantics, and severe incompleteness have lead modern modal logic away from science.

Kurpfalzstr. 53, 69 226 Nußloch, Germany
mueller-theys@gmx.de

Manat Mustafa

Elementary theories and hereditary undecidability for semilattices of numberings

joint work with Nikolay Bazhenov, Mars Yamaleev

The main motive in the study of degree structures of all kinds has been the question of the decidability or undecidability of their first order theories. This is a fundamental question that is an important goal in the analysing of these structures. A decision procedure implies and requires a full understanding and control of the first order properties of a structure[4]. In this talk, we will show undecidability for theories of upper semilattices that arises from the theory of numberings[1]. We use the following approach: given a level of complexity, say Σ^0_α, we consider the upper semilattice $R_{\Sigma^0_\alpha}$ of all Σ^0_α-computable numberings of all Σ^0_α-computable families of subsets of $\mathbb{N}$. We prove that the theory of the semilattice of all computable numberings is m-equivalent to the first order arithmetic. We show that the theory of the semilattice of all numberings is m-equivalent to the second order arithmetic. We also obtain a lower bound for the m-degree of the theory of the semilattice of all Σ^0_α-computable numberings, where $\alpha \geq 2$ is a computable successor ordinal. Furthermore, it is shown that for any of the theories T mentioned above, the Π_5-fragment of T is hereditarily undecidable. Similar results are obtained for the commutative monoid of all computably enumerable equivalence relations on $\mathbb{N}$, under composition.

References

[1] *Yu. L. Ershov*, Theory of numberings, Nauka, Moscow, 1977. -416 p. [In Russian].

[2] *S. S. Goncharov, A. Sorbi*, Generalized computable numerations and nontrivial Rogers semilattices, Algebra Logic, 36:6 (1997), 359–369.

Algebra Logic, 44:3 (2005), 143–147.

[3] *Yu. L. Ershov*, Problems of decidability and constructive models, Nauka, Moscow,1980.-415 p. [In Russian].

[4] *A. Nies*, Undecidable fragments of elementary theories, Algebra Univers., 35:1 (1996), 8–33.

Department of Mathematics,SST, Nazarbayev University,53, Kabanbay Batyr Avenue, Astana, 010000, Kazakhstan

manat.mustafa@nu.edu.kz

Satoru Niki

On some notions of negation between contraposition and minimal negation

The principle of *ex falso quodlibet* (EFQ) states that contradiction entails everything: $(A \wedge \neg A) \to B$. EFQ is valid in intuitionistic logic, but this is seen as problematic by Johansson, who created an alternative subsystem called minimal logic [3]. In minimal logic, EFQ is rejected albeit not entirely, as a weaker version $(A \wedge \neg A) \to \neg B$ still holds.

Some subsystems of minimal logic are investigated in Colacito [1] and Colacito et al. [2] in the framework of *subminimal logics*. The start point for them is the observation that minimal logic has a negation axiom if $\neg$ rather than $\bot$ is taken as primitive. Compared with minimal logic, subminimal logics have weaker negation axioms, while maintaining the positive axioms and rules. One logic they investigated was contraposition logic characterised by the axiom $(p \to q) \to (\neg q \to \neg p)$, Some subminimal logics were shown to be subsystems of contraposition logic, but it is not well-studied what subminimal logics have contraposition logic as a subsystem.

In the talk, I shall present some logics between contraposition logic and minimal logic, with a discussion of their proof-theoretic and semantic properties. In particular, I shall talk about logics which has a semantics that is a straightforward generalisation of the Kripke semantics for minimal logic.

References

[1] Almudena Colacito, ***Minimal and Subminimal Logic of Negation***, Master's Thesis, University of Amsterdam, 2016.

[2] Almudena Colacito, Dick de Jongh, Ana Lucia Vargas, *Subminimal negation*, ***Soft Computing***, vol. 21 (2017), pp. 165–174.

[3] Ingebrigt Johansson, *Der MinimalkalkÃijl, ein reduzierter intuitionistischer Formalismus*, ***Compositio Mathematica***, vol. 4 (1937), pp. 119–136.

School of Information Science, Japan Advanced Institute of Science and Technology, 1-1 Asahidai, Nomi, Ishikawa 923-1292, Japan.
s1610141@jaist.ac.jp

Cyrus F Nourani

Filters, language topologies, and product models

A Brief on Direct Product Languages and Models Languages $L_1, \dots, L_w$ with signatures S_i's, … , $S_i < S_{i+1}$. Applying inclusion ordering on the signatures $S*$ we have morphic preorders on the free trees on the signature TS_i.

Proposition 1. *There is a small complete category on the infinitary language fragments definable with the S_i's based on the direct product on TS_i.*

Consider the fragment topology defined on the author defined on Kiesler L^1_w, K fragments. Let us have a glimpse on n-types and positive local realizability (Nourani 2007-2015). The set of all complete n-types over T is denoted $n(T)$.

THEOREM 2 (Nourani 2014). *There is a generic functor on the category the omitting n-types realizing a direct product model.*

THEOREM 3. *The category based on L^1_w, K fragments and preorder morphisms is a convergent space category.*

References

[1] CYRUS F NOURANI AND PATRIK EKLUND, *Term Functors and Product Models: A Brief*, ***Joint MM AMS-MAM, Atlanta, Georgia meeting***, abstract 1125-VJ-2308 (2017).

Informatik, Ernest Reuter Platz,TU Berlin AI, Germany & SFU
cyrusfn@alum.mit.edu

SABRINA OUAZZANI

Computability and cheap non-standard analysis

joint work with Olivier Bournez

Non standard Analysis (NSA) is an area of Mathematics dealing with notions of infinitesimal and infinitely large numbers, in which many statements from classical Analysis can be expressed very naturally. Cheap NSA introduced by Terence Tao in 2012 [2] is based on the idea that considering that a property holds eventually is sufficient to give the essence of many of its statements. This provides constructibility but at some (acceptable) price.

We consider computability in cheap NSA. We prove that many concepts from computable analysis as well as several concepts from computability can be very elegantly and alternatively presented in this framework. For example, a standard real x is proved to be computable in the classical sense (of computable analysis) iff there exists some cheap non-standard computable rational $\frac{p}{q}$, such that $\left|x - \frac{p}{q}\right| \leq \epsilon$ for some effective infinitesimal ϵ. As another example, a function $f : [0,1] \to \mathbb{R}$

is computable in the classical sense (of computable analysis) iff it satisfies some discretization property (there exists some computable δ such that if $|x-y| \leq \delta$ then $|f(x)-f(y)| \leq \epsilon$.) and it has some uniform approximation function over the rationals.

We illustrate on various statements from analysis and computable analysis that cheap NSA provides a dual view and elegant dual proofs to several statements already known in these fields.

We will also discuss some relations between cheap non-standard analysis and computations with an ordinal time: In particular, considering objects indexed by an ordinal greater than omega allows to discuss effectiveness and complexity of mathematical statements for functions over the reals in higher levels of descriptive set theory than classical computable analysis. This provides ways to deal with functions in hierarchies in the spirit of Kechrin and Woodin for continuous non-differentiable functions, covering both boldface [1], and lightface versions of these hierarchies in the spirit of [3].

References

[1] Alexander S Kechris and W Hugh Woodin. Ranks of differentiable functions. *Mathematika*, 33(2):252–278, 1986.

[2] Terence Tao, April 2012.

[3] Linda Brown Westrick. A lightface analysis of the differentiability rank. *The Journal of Symbolic Logic*, 79(1):240–265, 2014.

LACL, Université Paris-Est Créteil, 61 avenue du Général de Gaulle, 94010 Créteil, France
sabrina.ouazzani@lacl.fr

Fedor Pakhomov

Reflection ranks and proof theoretic ordinals

joint work with James Walsh

It is a well-known phenomenon that natural theories are prewellordered with respect to the consistency strength order. But there are (artificial) examples of descending chains of theories in this order. We propose an explanation of this phenomenon by considering coarser order of Π^1_1-reflection strength: $T \prec_{\Pi^1_1} U \overset{\text{def}}{\iff} U$ proves Π^1_1-soundness of T. Note that in many cases natural consistency proofs moreover give

Π^1_1-soundness. We show that the restriction of the order to Π^1_1-sound extensions of ACA_0 is well-founded.

This well-foundedness result allows us to consider the notion of reflection rank for Π^1_1-sound extensions of ACA_0 (i.e. well-founded rank with respect to the order $\prec_{\Pi^1_1}$). We show that for extensions of ACA_0^+ the reflection rank of a theory coincides with the Π^1_1 proof-theoretic ordinal of the theory. In general, for extensions of ACA_0 if the Π^1_1 proof-theoretic ordinal is ε_α then the Π^1_1 reflection rank is $\leq \alpha$. In the case of theories of α-times iterated Π^1_1 reflection we show that Π^1_1 proof-theoretic ordinal is exactly ε_α.

We also show that our well-foundedness result as a direct corollary provides well-foundedness of Beklemishev's ordinal notation system up to ε_0 [1].

References

[1] Lev D. Beklemishev, *Provability algebras and proof-theoretic ordinals*, ***Annals of Pure and Applied Logic***, vol. 128 (2004), iss. 1–3, pp. 103–123.

Steklov Mathematical Institute, Moscow
pakhfn@mi.ras.ru

Gianluca Paolini

Model Theory of Free Projective Planes

joint work with Tapani Hyttinen

We prove that the theory of open projective planes is complete and strictly stable. As a corollary, we prove that Marshall Hall's free projective planes ($\pi^n : 4 \leq n \leq \omega$) are all elementary equivalent and that their common theory is strictly stable and decidable (being in fact the theory of open projective planes).

Einstein Institute of Mathematics, The Hebrew University of Jerusalem, Israel
gianluca.paolini@mail.huji.ac.il

Franco Parlamento

Absorbing the Structural Rules in the Sequent Calculus with Additional Atomic Rules

joint work with Flavio Previale

A multisuccedent sequent calculus for intuitionistic logic free of structural rules was presented in [2] and a detailed proof of their admissibility, based on [1], appeared in [5]. A single succedent version of that calculus was adopted in [6]. In all cases the proof of the admissibility of the structural rules relies, as for the classical **G3** system, on the hight-preserving admissibility of the contraction rule, that, when atomic rules are added to the calculus, may fail. Letting **G3**[**mic**] be the slight variants described in [3] for minimal, intuitionistic and classical logic of the calculi in [5], and $\mathcal{R}$ any set of atomic rules of the following form:

$$\frac{\vec{Q}_1, \Gamma_1 \to \Delta_1, \vec{Q'_1} \quad \ldots \quad \vec{Q}_n, \Gamma_n \to \Delta_n, \vec{Q'_n}}{\vec{P}, \Gamma_1, \ldots, \Gamma_n \to \Delta_1, \ldots, \Delta_n, \vec{P'}}$$

where $\vec{Q}_1, \vec{Q'_1}, \ldots, \vec{Q}_n, \vec{Q'_n}, \vec{P}, \vec{P'}$ are sequences (possibly empty) of atomic formulae and $\Gamma_1, \ldots, \Gamma_n, \Delta_1, \ldots \Delta_n$ are finite sequences (possibly empty) of formulae that are not active in the rule, and letting $\mathbf{G3}[\mathbf{mic}]^{\mathcal{R}}$ denote the calculi obtained by adding to **G3**[**mic**] the rules in $\mathcal{R}$ and Cut_{cs} the contex-sharing cut rule, we have the following:

Theorem 1. *For any set of atomic rules $\mathcal{R}$, any derivation in $\mathbf{G3}[\mathbf{mic}]^{\mathcal{R}} + Cut_{cs}$ can be transformed into a derivation in the same system in which the rules in $\mathcal{R}$ and the Cut_{cs} rule are applied before any logical rule.*

Corollary 2. *If the structural rules are admissible in the calculus that contains only the initial sequents and the rules in $\mathcal{R}$, then they are admissible in $\mathbf{G3}[\mathbf{mic}]^{\mathcal{R}}$ as well.*

Two applications are the following. 1) For $\mathcal{R} = \emptyset$ we have a proof of the admissibility of the *Cut* rule in **G3**[**mic**] independent from the hight-preserving admissibility of the contraction rule. 2) Let Ref and Repl be the following rules for equality, introduced in [4] and adopted in the second edition [7] of [6]

$$\frac{t = t, \Gamma \to \Delta}{\Gamma \to \Delta}\ \text{Ref} \qquad\qquad \frac{s = r, P[x/s], P[x/r], \Gamma \to \Delta}{s = r, P[x/s], \Gamma \to \Delta}\ \text{Repl}$$

The contraction rule is not height preserving admissible in $\mathcal{R} = \{\mathrm{Ref}, \mathrm{Repl}\}$, yet we obtain that the structural rules are admissible in $\mathbf{G3}[\mathbf{mic}]^{\mathcal{R}}$, thus extending the result proved in [4] in the case t, r and s are restricted to be constants.

References

[1] R. Dyckhoff, *Dragalin's proof of cut-admissibility for the intuitionistic sequent calculi* **G3i** *and* **G3i'**, ***Research Report CS/97/8 - Computer Science Division*** , St. Andrews University (1997)

[2] A. Dragalin, *Mathematical Intuitionism:Introduction to Proof Theory*, ***Translation of mathematical monographs 67*** , *Russian original of 1979 , American Mathematical Society (1988)*

[3] S. Negri, *Glivenko sequent classes in the light of structural proof theory*, ***Arch. Math. Logic*** 55, 461-473 (2016)

[4] J. von Plato, S. Negri, *Cut Elimination in the Presence of Axioms*, ***The Bulletin of Symbolic Logic*** , 4 (4) , 418–435 (1998)

[5] J. von Plato, S. Negri, *Structural Proof Theory*, Cambridge University Press (2001)

[6] A.S. Troelstra, H. Schwichtemberg, *Basic Proof Theory Cambridge Tracts in Theoretical Computer Science, Vol. 43* Cambridge University Press, Cambridge(1996).

[7] A.S. Troelstra, H. Schwichtemberg, *Basic Proof Theory. 2nd edition* Cambridge University Press, Cambridge(2000).

Department of Mathematics, Computer Science and Physics, University of Udine, via delle Scienze 206, 33100 Udine, Italy

`franco.parlamento@uniud.it`

Luiz Carlos Pereira

Gentzen's second consistency proof and Gödel's koan

joint work with Edward Haeusler and Daniel Durante

In 1938 Gentzen published a new proof of the consistency of elementary arithmetic ([2]). It is known that the reductions used by Gentzen in this proof can be used to obtain cut-elimination for Gentzen‘s LK and LJ, and strong cut-elimination for the propositional fragment of LK. In 1968 William Howard proposed an assignment of ordinals $< \epsilon_0$ to terms for primitive recursive functionals of finite type such that reductions always lowers the ordinal measure. Howard's assignment motivated the appearence in 2014 of problem 26 in the TLCA list of open problems,

the so-called Gödel's Koan.
"Statement: Assign (in an "easy" way) ordinals to terms of the simply typed lambda calculus such that reduction of the term yields a smaller ordinal."
The aim of this paper is to show that Gentzen's proof can contribute to a solution to Gödel's koan. We will show that Gentzen's ordinal assignment produces "easy" proofs of strong cut-elimination for the system FIL (see [1]) and of strong normalization for the implicational fragment of the usual Natural deduction for intuitionistic logic.

References

[1] de Paiva, Valeria and Pereira, Luiz C., A short note on intuitionistic propositional logic with multiple conclusions, in *Manuscrito.*, Campinas, v. 28, n. 2, p. 317-329, 2005.

[2] Gentzen, Gerhard, Neue Fassung des Wiederpruchfreiheit-beweises für die Reine Zahlentheorie, in *Forschung zur Logik und zur Grundlegung der exakten Wissenschaften*, Neue Hefte 4, Leiptzig, 1938.

[3] Howard, William A., Assignment of ordinals to primitive recursive functionals of finite type, in *Intutionism and Proof Theory*, (eds) A. Kino, J. Myhill and R. E. Vesley, North-Holland, Amsterdam, pp. 443 - 458,1970

Department of Philosophy - PUC-Rio/UERJ
luiz@inf.puc-rio.br

Garik Petrosyan

On the relations between the proof complexity measures of strongly equal k-tautologies in some proof systems

joint work with Anahit Chubaryan

In the mean time many interesting applications of many-valued logic (MVL) were found in such fields as Logic, Mathematics, Formal Verification, Artificial Intelligence, Operations Research, Computational Biology, Cryptography, Data Mining, Machine Learning, Hardware Design etc., therefore the investigations of proof complexity for different systems of MVL are very important.

The traditional assumption that all tautologies as Boolean functions are equal to each other is not fine-grained enough to support a sharp distinction among tautologies. The authors of [1] have provided a different picture of equality for classical tautologies. They have introduced the notion of strong equality of 2-valued tautologies on the basis of determinative conjunct notion. The idea to revise the notion of equivalence between tautologies in such way that is takes into account an appropriate measure of their "complexity". It was proved in [2,3] that in "weak" proof systems the strongly equal 2-valued tautologies have the same proof complexities, while in the "strong" proof systems the measures of proof complexities for strongly equal tautologies can essentially differ from each other. Here we generalize the notions of determinative conjunct and strongly equal tautologies for MVL and compare the proof complexity measures of **strongly equal many-valued tautologies** in some proof systems of MVL. We prove the following statement.

Theorem. *There are the sequences of strongly equal k-valued ($k \geqslant 3$) tautologies A_n and B_n such, that a) the lengths and sizes of A_n and B_n proofs in some "weak" systems of MVL are the same, b) if $\mathbf{\Phi}$ is one of "strong" systems of MVL, then*

$$\boldsymbol{t^{\Phi}_{A_n} = O(1), \quad l^{\Phi}_{A_n} = O(n),}$$
$$\boldsymbol{t^{\Phi}_{B_n} = \Omega(n), \quad l^{\Phi}_{B_n} = \Omega(n^2),}$$

where $\boldsymbol{t^{\Phi}_{A_n}}$ ($\boldsymbol{t^{\Phi}_{B_n}}$) and $\boldsymbol{l^{\Phi}_{A_n}}$ ($\boldsymbol{l^{\Phi}_{B_n}}$) are the minimal measures of $\mathbf{\Phi}$-proofs lengths and sizes for formulas $\boldsymbol{A_n}$ ($\boldsymbol{B_n}$).

References

[1] An. Chubaryan, Arm. Chubaryan, *A new conception of Equality of Tautologies*, ***L&PS, Triest, Italy***, Vol. V, No 1, 2007, 3–8.

[2] A. Chubaryan, G. Petrosyan, *The relations between the proof complexities of strongly equal classical tautologies in Frege systems*, **The Russian-Chinese Scientific Journal "Commonwealth", Phiz-Mat Science**, No. I (1), 2016, 78–80.

[3] An. Chubaryan, A. Mnatsakanyan, *On the bounds of the main proof measures in some propositional roof systems*, ***Scholar Journal of Phis. Math. And Stat.***, Vol. 1, Issue-2, 2014, 111-117.

Department of Informatics and Applied Mathematics, Yerevan State University, 1 Alex Manoogian, Armenia
garik.petrosyan.1@gmail.com

ALBERTO POLICRITI

Encoding Sets as Real Numbers

joint work with Domenico Cantone

In 1937, W. Ackermann proposed the following encoding of hereditarily finite sets by natural numbers:

(1) $$\mathbb{N}_A(x) := \sum_{y \in x} 2^{\mathbb{N}_A(y)}.$$

The encoding $\mathbb{N}_A$ is simple, elegant, and highly expressive for a number of reasons. On the one hand, it builds a strong bridge between two foundational mathematical structures: (hereditarily finite) sets and (natural) numbers. On the other hand, it enables the representation of the *characteristic function* of hereditarily finite sets in terms of the usual notation for natural numbers by sequences of binary digits. That is: y belongs to x if and only if the $\mathbb{N}_A(y)$-th digit in the binary expansion of $\mathbb{N}_A(x)$ is equal to 1. As one would expect, the string of 0's and 1's representing $\mathbb{N}_A(x)$ is nothing but (a representation of) the characteristic function of x.

We study a very simple variation of the encoding $\mathbb{N}_A$, applicable to a larger collection of sets. The proposed variation is obtained by simply putting a minus sign before each exponent in 1 resulting in the expression:

(2) $$\mathbb{R}_A(x) := \sum_{y \in x} 2^{-\mathbb{R}_A(y)}.$$

The range of $\mathbb{R}_A$ is now a collection of *real* numbers and its domain can be enlarged to include *non-well-founded* hereditarily finite sets, that is sets defined by (finite) systems of equations of the following form

(3) $$\begin{cases} x_1 &= \{x_{1,1}, \ldots, x_{1,m_1}\} \\ x_2 &= \{x_{2,1}, \ldots, x_{2,m_2}\} \\ \vdots & \qquad \vdots \\ x_n &= \{x_{n,1}, \ldots, x_{n,m_n}\}, \end{cases}$$

with *bisimilarity* as equality criterion (see [Acz88] and [BM96], where the term *hyperset* is also used). For instance, the special case of the single equation $x_1 = \{x_1\}$, resulting in the equation $x = 2^{-x}$, provides the code of the unique (under bisimilarity) hyperset $\Omega = \{\Omega\}$.[1]

[1] Notice that the solution to the equation $x = e^{-x}$ is the so-called *omega* constant, introduced by Lambert in [Lam58] and studied also by Euler in [Eul83].

As a matter of fact, $\mathbb{N}_A$ is defined inductively and this is perfectly in line with our intuition on the very basic properties of the collection of natural numbers $\mathbb{N}$ and the collection of hereditarily finite sets HF—called HF^0 in [BM96]. The definition of $\mathbb{R}_A$, instead, is not inductive and it requires a more careful analysis, as it must be *proved* to injectively associate (real) numbers to sets.

The injectivity of $\mathbb{R}_A$ on the collection of non-well-founded sets—henceforth, to be referred to as $\mathsf{HF}^{1/2}$, see [BM96]—was conjectured in [Pol13] and is still an open problem. Here we prove that, given any finite collection $\hbar_1, \ldots, \hbar_n$ of elements of $\mathsf{HF}^{1/2}$ satisfying a minimal system of set-theoretic equations of the form (3) in the variables $x_1, \ldots, x_n$, we can univocally determine real numbers $\mathbb{R}_A(\hbar_1), \ldots, \mathbb{R}_A(\hbar_n)$ satisfying the following system of equations:

$$\begin{cases} \mathbb{R}_A(\hbar_1) & = & \sum_{k=1}^{m_1} 2^{-\mathbb{R}_A(\hbar_{1,k})} \\ \mathbb{R}_A(\hbar_2) & = & \sum_{k=1}^{m_2} 2^{-\mathbb{R}_A(\hbar_{2,k})} \\ \quad\vdots & & \quad\vdots \\ \mathbb{R}_A(\hbar_n) & = & \sum_{k=1}^{m_n} 2^{-\mathbb{R}_A(\hbar_{n,k})}. \end{cases}$$

This preliminary result shows that the definition of $\mathbb{R}_A$ is well-given, as it associates a unique (real) number to evey non-well-founded hereditarily finite set. This extends to $\mathsf{HF}^{1/2}$ the first of the properties that the encoding $\mathbb{N}_A$ enjoys with respect to HF. Should $\mathbb{R}_A$ also enjoy the injectivity property, our proposed adaptation of N would be completely satisfiying, and $\mathbb{R}_A$ could be coherently dubbed an *encoding* for non-well-founded sets.

In the course of our proof, we shall also introduce a procedure that drives us to the real numbers $\mathbb{R}_A(\hbar_1), \ldots, \mathbb{R}_A(\hbar_n)$ mentioned above by way of successive approximations. In the well-founded case, our procedure will converge in a finite number of steps, whereas, in the non-well-founded case, infinitely many steps will be required for convergence. In the last part of the paper, we shall also briefly discuss some computational issues related with the problem of computing approximations of the values $\mathbb{R}_A(\hbar)$.

References

[Ack37]W. Ackermann, *Die Widerspruchfreiheit der allgemeinen Mengenlehre*, Mathematische Annalen **114** (1937), 305–315.

[Acz88]P. Aczel, *Non-well-founded sets*, vol. 14 of CSLI Lecture Notes, Stanford, CA, 1988.

[BM96]J. Barwise and L. S. Moss, *Vicious circles*, CSLI Lecture Notes, Stanford, CA, 1996.

[Eul83]L. Euler, *De serie Lambertina Plurimisque eius insignibus proprietatibus.*, Acta Acad. Scient. Petropol. **2** (1783), 29–51, reprinted in Euler, L. Opera Omnia, Series Prima, Vol. 6: Commentationes Algebraicae. Leipzig, Germany: Teubner, pp. 350–369, 1921.

[Lam58]J.H. Lambert, *Observations variae in Mathesin Puram*, Acta Helvitica, physico-mathematico-anatomico-botanico-medica. **3** (1758), 128–168.

[Pol13]Alberto Policriti, *Encodings of sets and hypersets*, Proceedings of the 28th Italian Conference on Computational Logic, Catania, Italy, September 25-27, 2013., 2013, pp. 235–240.

Dipartimento di Scienze Matematiche, Informatiche e Fisiche, University of Udine, Italy
alberto.policriti@uniud.it

Michele Pra Baldi

Logics of variable inclusion and Płonka sums of matrices

joint work with Stefano Bonzio and Tommaso Moraschini

It is always possible to associate with an arbitrary propositional logic $\vdash$, two new substitution-invariant consequence relations $\vdash^l$ and $\vdash^r$, which satisfies respectively a *left* and a *right variable inclusion principle*, as follows:

$$\Gamma \vdash^l \varphi \iff \text{there is } \Delta \subseteq \Gamma \text{ s.t. } \mathsf{Var}(\Delta) \subseteq \mathrm{Var}(\varphi) \text{ and } \Delta \vdash \varphi,$$

and

$$\Gamma \vdash^r \varphi \iff \begin{cases} \Gamma \vdash \varphi \text{ and } \mathsf{Var}(\varphi) \subseteq \mathsf{Var}(\Gamma) & \text{or} \\ \Sigma \subseteq \Gamma \end{cases}$$

where Σ is a set of inconsistency terms for $\vdash$. Accordingly, we say that the logics $\vdash^l$ and $\vdash^r$ are respectively the *left* and the *right variable inclusion companions* of $\vdash$.

Prototypical examples of variable inclusion companions are found in the realm of three-valued logics. For instance, the left and the right variable inclusion companions of classical (propositional) logic are respectively *paraconsistent weak Kleene logic* (PWK for short) [4], and *Bochvar logic* [1].

Recent work [2] linked PWK to the algebraic theory of regular varieties, i.e. equational classes axiomatized by equations $\varphi \approx \psi$ such that

$\mathsf{Var}(\varphi) = \mathsf{Var}(\psi)$. The representation theory iof regular varieties is largely due to the pioneering work of Płonka [5], and is tightly related to a special class-operator $\mathcal{P}(\cdot)$ nowadays called *Płonka sums*. This observations led us to investigate the relations between left and right variable inclusion companions and Płonka sums in full generality. Our study is carried on in the conceptual framework of abstract algebraic logic [3, 6].

At first, we define an appropriate notion of *direct system* of logical matrices, and we lift the construction of Płonka sums from algebras to logical matrices. This new technique allows to provide a completeness theorem for arbitrary logics of variable inclusion $(\vdash^l, \vdash^r)$ by performing Płonka sums over direct systems of models of $\vdash$. Then, we present a general method to transform every Hilbert-style calculus for a finitary logic $\vdash$ with a *partition function* into complete Hilbert-style calculi for $\vdash^l$ and $\vdash^r$ respectively. Moreover, we describe the structure of the matrix semantics $\mathsf{Mod}^{\mathsf{Su}}(\vdash^l), \mathsf{Mod}^{\mathsf{Su}}(\vdash^r)$, given by the so-called Suszko reduced models of $\vdash^l, \vdash^r$. We close our investigation by determining the location of logics of variable inclusion in the Leibniz hierarchy.

References

[1] D. Bochvar, *On a three-valued calculus and its application in the analysis of the paradoxes of the extended functional calculus*, ***Mathematicheskii Sbornik***, 4:287–308, 1938.

[2] S. Bonzio, J. Gil-Férez, F. Paoli, and L. Peruzzi, *On Paraconsistent Weak Kleene Logic: axiomatization and algebraic analysis*, ***Studia Logica***, 105(2):253–297, 2017.

[3] J. Font, ***Abstract Algebraic Logic: An Introductory Textbook***, College Publications, 2016.

[4] S. Halldén, *The Logic of Nonsense*, ***Lundequista Bokhandeln***, Uppsala, 1949.

[5] . Płonka, *On a method of construction of abstract algebras*, ***Fundamenta Mathematicae***, 61(2):183–189, 1967.

[6] J. Czelakowski, ***Protoalgebraic logics***, volume 10 of Trends in Logic, Studia Logica Library, Kluwer Academic Publishers, Dordrecht, 2001.

Università di Padova, Padova, Italy

Alexander Rabinovich

On definability of Kinna-Wagner choice over trees

A Kinna-Wagner choice function assigns to every set of cardinality at least two its non-empty proper subset. A Kinna-Wagner choice axiom is strictly weaker than Axiom of Choice, it implies that every set can be linearly ordered [3, 2], but does not imply that every set can be well-ordered.

We deal with the following definability question: For what trees $\mathcal{T} := (T, \leq)$ it is the case that there is a finite sequence $\bar{P}$ of subsets of T and a formula $\varphi(X, Y, \bar{Z})$ of monadic second-order (MSO) logic such that

1. $\mathcal{T} \models$ "if X has at least two elements then $\exists Y \varphi(X, Y, \bar{P})$"
2. $\mathcal{T} \models \varphi(X, Y, \bar{P}) \to (X \supset Y \neq \emptyset)$
3. $\mathcal{T} \models \big(\varphi(X, Y_1, \bar{P}) \wedge \varphi(X, Y_2, \bar{P})\big) \to Y_1 = Y_2$

When the answer to this question is positive we say that φ defines a KW choice function on $\mathcal{T}$ and $\mathcal{T}$ has an MSO definable (with parameters) KW choice function.

Theorem 1. *Let $\mathcal{T}$ be a tree. The following are equivalent:*

1. *There is an MSO formula that defines with parameters a KW choice function on $\mathcal{T}$.*
2. *There is an MSO formula $\psi(x, y, \bar{Z})$ that defines with parameters a linear order on $\mathcal{T}$.*
3. *$\mathcal{T}$ has branching (out-degree) at most k for some $k \in \mathbb{N}$.*

This theorem is a variant of much more subtle results in [1, 4], where trees with definable choice function were characterized. The implications (3)→(1) and (3)→(2) are obtained by an easy formalization. The proof of implications (1)→(3) and (2)→(3) uses compositional theorems for monadic logic over trees [1, 4].

References

[1] Y. Gurevich and S. Shelah. *Rabin's uniformization problem.* J. Symbolic Logic, 48(4):1105-1119, 1983.

[2] H. Herrlich. Axiom of Choice. Lecture Notes in Mathematics 1876, 2006.

[3] W. Kinna and K. Wagner. *Über eine Abschwächung des Auswahlpostulates.* Fund. Math., 42:75-82, 1955.

[4] S. Lifsches and S. Shelah. *Uniformization, choice functions and well orders in the class of trees.* J. Symbolic Logic, 61(4):1206-1227, 1996.

Tel Aviv University
rabinoa@post.tau.ac.il

Christian Retoré

Individuals, equivalences and quotients in type theoretical semantics

joint work with Léo Zaradzki

In natural language semantics, individuation and the nature of individuals is highly debated. If one says *I carried all the books from the shelf to the cellar because I read them all* and if there were five books, two of them being *Dubliners*, five books were carried, while four were read. This raises the question whether individuals are the same or not, and type theoretical semantics with equality types offer new perspectives on this question.

Mathematically, sameness corresponds to equivalence: equivalent objects enjoy some common properties, their equivalence classes can be proved to be equal. Equivalence can be coarser or finer grained: a book may be defined as a novel, as an edition of this novel or as a particular copy of some particular edition.

We show how linguistically motivated quotient constructions can be integrated in type theoretical semantics, with insights from some recent work on quotients in proof assistants — indeed both cases require to only encode computable quotients.

References

[1] Stergios Chatzikyriakidis, and Zhaohui Luo, editors. ***Modern Perspectives in Type Theoretical Semantics***, Springer, Heidelberg, 2017.

[2] Cyril Cohen, *Pragmatic quotient types in Coq*, ***ITP2013 International Conference on Interactive Theorem Proving*** (Rennes), (Sandrine Blazy, Christine Paulin-Mohring, David Pichardie, editors), vol. 7998, Springer, 2013, pp. 213–228.

[3] Christian Retoré, *The Montagovian Generative Lexicon ΛTy_n: a Type Theoretical Framework for Natural Language Semantics*, ***19th International Conference on Types for Proofs and Programs (TYPES 2013)*** (Toulouse), (Ralph Matthes and Aleksy Schubert, editors), vol. 26, Leibniz International Proceedings in Informatics (LIPIcs), 2014, pp. 202–229.

Univ. Montpellier & LIRMM, 161 rue Ada, 34090 Montpellier, France
christian.retore@umontpellier.fr

ABILIO RODRIGUES

An epistemic approach to paraconsistency: a logic of evidence and truth

joint work with Walter Carnielli

The basic idea of the epistemic approach to paraconsistency here proposed is that logic is not restricted to preservation of truth. Classical logic gives an account of strict preservation of truth, but sometimes truth is not the only property of propositions that is at stake in argumentative contexts. So, the acceptance of a pair of contradictory propositions A and $\neg A$ does not commit us to their truth. Rather, we understand it as some kind of conflicting information about A, namely, that there is *conflicting evidence* about A. Evidence for A (resp. $\neg A$) is understood as reasons for believing in and/or accepting A as true (resp, false). Evidence is clearly a notion weaker than truth, in the sense that there may be evidence for the truth of a proposition A even if A is, in fact, false, and will be eventually established later as false.

We introduce a natural deduction system called the Basic Logic of Evidence (BLE) designed to express preservation of evidence instead of preservation of truth. The system is paraconsistent and paracomplete, since neither explosion nor excluded middle hold. The Logic of Evidence and Truth (LET_j), a logic of formal inconsistency and undeterminedness, is obtained by extending BLE with a recovery operator and rules that restore the validity of excluded middle and explosion, and so classical logic, for propositions whose truth value has already been conclusively established. Once classical logic is recovered, the system turns out to be able to give an account of both preservation of evidence and preservation of truth.

Adequate valuation semantics and decision procedures for BLE and LET_j are provided. These semantics, however, are not intended to have an intuitive appeal independent of the deductive system. Rather, they should be seen as mathematical tools capable of representing the inference rules in such a way that technical results can be proved. In order to give meanings to the connectives of BLE and LET_j we extend the basic idea of inferential (or proof-theoretic) semantics. The meanings of the connectives of BLE are given by the inference rules, but in a context where preservation of evidence is at stake. Analogously, in LET_j, the meaning of the connectives is given by the allowed inferences, depending on what is being preserved, truth or evidence.

References

[1] W. Carnielli and A. Rodrigues, *An epistemic approach to paraconsistency: a logic of evidence and truth*, ***Synthese*** (2017), https://doi.org/10.1007/s11229-017-1621-7.

Department of Philosophy, Federal University of Minas Gerais, 31270-901, Brazil
abilio.arf@gmail.com

Dino Rossegger

Computable bi-embeddable categoricity

joint work with Nikolay Bazhenov, Ekaterina Fokina, and Luca San Mauro

The study of morphisms between computable structures is a main theme in computable structure theory since Fröhlich and Shepherdson, and Mal'cev exhibited computable structures that are not computably isomorphic.

We study the complexity of embeddings between bi-embeddable computable structures. To this end we define and investigate the notions *(relative) b.e. categoricity* and *degree of b.e. categoricity*. These notions are analogues to the well studied notions for isomorphisms between computable structures. As general results we obtain that every Turing degree $\mathbf{d}$ that is d-c.e. above $\mathbf{0}^{(\alpha)}$ for α a computable successor ordinal is the degree of b.e. categoricity of a structure and that $\mathbf{0}'$ b.e. categoricity is Π^1_1 complete.

We furthermore prove that every computable equivalence structure has degree of categoricity $\mathbf{0}$, $\mathbf{0}'$, or $\mathbf{0}''$ and characterize the computable b.e. categorical structures in several natural classes of structures.

References

[1] Nikolay Bazhenov, Ekaterina Fokina, Dino Rossegger, Luca San Mauro, *Degrees of bi-embeddable categoricity of equivalence structures*, ***submitted for publication***.

[2] Nikolay Bazhenov, Ekaterina Fokina, Dino Rossegger, Luca San Mauro, *Computable bi-embeddable categoricity*, ***to appear in Algebra and Logic***.

Institute of Discrete Mathematics and Geometry, Vienna University of Technology, Wiedner Hauptstraße 8-10, 1040 Wien, Austria
dino.rosseger@tuwien.ac.at

Luca San Mauro

Learning equivalence structures

joint work with Ekaterina Fokina and Timo Kötzing

Algorithmic learning theory is a vast research program, initiated by Gold [1] in the 1960's, that comprises different models of learning in the limit. It deals with the question of how a learner, provided with more and more data about some environment, is eventually able to achieve systematic knowledge about it.

In this paper, we investigate computable structures through the lens of algorithmic learning theory (see, e.g., [2] for another study in this direction). We introduce the following framework. Let $\mathbb{K}$ be a class of structures. Suppose we have an effective numbering $\mathcal{A}_0, \mathcal{A}_1, \dots, \mathcal{A}_i, \dots$ of computable structures from $\mathbb{K}$. Let M be a partial computable function which takes for its inputs finite substructures of a structure $\mathcal{A}$ from the class $\mathbb{K}$ (where $\mathcal{A}^0 \subseteq \mathcal{A}^1 \subseteq \dots \subseteq \mathcal{A}^i \subseteq \dots$ and $\mathcal{A} = \bigcup_i \mathcal{A}^i$) and which either goes undefined or returns a number of program. For a finite initial substructure $\mathcal{A}^i$, if $M(\mathcal{A}^i) \downarrow = n$, then n represents M's conjecture or hypothesis as to an index for $\mathcal{A}$ in the above-mentioned numbering. The learner M $\mathbf{EX}_{\cong}$*-learns* $\mathcal{A}$ if there exists a number n such that $\mathcal{A} \cong \mathcal{A}_n$ and $M(\mathcal{A}^i) \downarrow = n$, for all but finitely many i. A family of structures $\mathfrak{A}$ is $\mathbf{EX}_{\cong}$-learnable if there is M that learns all $\mathcal{A} \in \mathfrak{A}$.

We focus on equivalence structures and study which families of equivalence structures with domain ω are $\mathbf{EX}_{\cong}$*-learnable*. We also unveil a natural hierarchy of different notions of learnability by replacing isomorphism with other relations expressing structural similarity, such as bi-embeddability ad bi-homomorphism.

References

[1] M. E. Gold, *Language identification in the limit*, ***Information and control***, 10.5 (1967), pp. 447–474.

[2] V. Harizanov, F. Stephan *On the learnability of vector spaces*, ***Journal of Computer and System Sciences***, 73.1 (2007), pp. 109–122.

Institute of Discrete Mathematics and Geometry, Vienna University of Technology
luca.san.mauro@tuwien.ac.at

Denis I. Saveliev

Higher indescribability

This work continues Beklemishev's work [2], in which derived topologies and corresponding generalizations of stationary sets were introduced and investigated. The starting point of these studies lies rather in proof theory than in set theory. Esakia's pioneer work [1] connected scattered spaces to Gödel–Löb's logic GL. This put beginning of studies of topological completeness of its extension, Japaridze's provability logic GLP. Solving the problem, Beklemishev introduced [2] the following construction: given a topology τ on a set X, the *derived* (or *next*) topology τ^+ on X is generated by τ and the sets $d(A)$ for all $A \subseteq X$, where $d(A)$ is the Cantor derivative of A given by τ, i.e., the set of τ-limit points of A. He considered ω-sequences of topologies defined by letting $\tau_0 = \tau$ and $\tau_{n+1} = \tau_n^+$. If τ_0 is the left (or Alexandroff) topology on an ordinal θ, then τ_1 is the standard interval topology of θ, τ_2 is the so-called club topology whose limit points are ordinals $\alpha < \theta$ of uncountable cofinality and $d_2(A) = \{\alpha < \theta : A \cap \alpha \text{ is stationary in } \alpha\}$, and τ_3 is called the Mahlo topology. It was shown [2] that τ_{n+2} is included into the topology $\tau_{\Pi^1_n}$ whose limit points are Π^1_n-indescribable ordinals, so a weakly compact suffices to have τ_3 non-discrete, and later [3] that in L (the constructible universe), τ_{n+2} and $\tau_{\Pi^1_n}$ coincide. An overview with some additional information can be found in [4].

We consider sequences of derived topologies of arbitrary length by letting $\tau_0 = \tau$, $\tau_{\alpha+1} = \tau_\alpha^+$, and τ_α = the topology generated by all τ_β, $\beta < \alpha$, if α is a limit ordinal. We show that, for $\alpha \geq \omega$, under an appropriate definition of Π^1_α-formulas of second-order infinitary languages, τ_α is included into $\tau_{\Pi^1_\alpha}$. Evaluating size of the least ordinals indescribable in higher-order infinitary languages, we show e.g. that if κ is the least Π^1_n-indescribable in $\mathcal{L}^2_{\alpha,\alpha}$ (the second-order language with connectives and quantifiers of arity $< \alpha$), then it is so already in $\mathcal{L}^2_{\omega,\omega}$ (the second-order finitary language). Furthermore, if θ is the ω-Erdős cardinal then the set $\{\alpha < \theta : \alpha \text{ is } \mathcal{L}^\alpha_{\alpha,\alpha}\text{-indescribable}\}$ is stationary in θ, and if θ is a measurable cardinal and U a normal ultrafilter on θ then this set is in U, therefore, almost all $\alpha < \theta$ are τ_α-limit points. The hierarchy of derived topologies on θ extends up to θ^+ by diagonalizing: for $\xi < \theta^+$ define $d_\xi(A) = \{\alpha < \theta : \alpha \in d_{\xi_\alpha}(A)\}$ if $\operatorname{cf} \xi = \theta$ with $\lim_{\alpha \to \theta} \xi_\alpha = \xi$, and as before otherwise. If θ is measurable then all the topologies τ_ξ also are non-discrete.

References

[1] L. Esakia, *Diagonal constructions, Löb formula and Cantor's scattered space*, **Studies in logic and semantics**, Metsniereba, Tbilisi, 1981, pp. 128–143 (in Russian).

[2] L. D. Beklemishev, *On GLP spaces*, Circulated manuscript, 2009.

[3] J. Bagaria, M. Magidor, H. Sakai, *Reflection and indescribability in the constructible universe*, ***Israel Journal of Mathematics***, vol. 208 (2015), no. 1, pp. 1–11.

[4] L. D. Beklemishev, D. Gabelaia, *Topological interpretations of provability logic*, **Leo Esakia on Duality in Modal and Intuitionistic Logics**, **Outstanding Contributions to Logic**, 4, 2014, pp. 257–290.

Steklov Mathematical Institute, Russian Academy of Sciences
d.i.saveliev@gmail.com

Ibrahim Senturk

Regarding Aristotelian Logic as a Sheffer Stroke Basic Algebra

joint work with Tahsin Oner

In this work, our aim is to tackle with the Aristotelian logic by means of Carroll's diagrammatic method. For this purpose, we firstly define a formal system which gives us a formal approach to logical reasoning with diagrams for representations of the fundamental propositions in Aristotelian logic. In the sequel, we show that syllogisms are closed under the syllogistic criterion of inference which is the deletion of middle term. Therefore, it allows for the formalism which comprise synchronically bilateral and trilateral diagrammatical appearance and a naive algorithmic nature. And also, there is no specific knowledge or exclusive ability needed in order to understand and use it.

In other perspective, we give a morphism from categorical syllogistic system to a Sheffer stroke basic algebra. To this end, we give a reduction of basic algebras by using only Sheffer stroke operation. Thereupon, we explain quantitative relation between two terms by means of bilateral diagrams. So, we obtain possible conclusion values of bilateral diagrams of premises. Finally, by using these sets, we construct a complete bridge between Sheffer stroke basic algebra and categorical logical system.

References

[1] Lewis Caroll. ***Symbolic Logic***, Clarkson N. Potter, 1896.

[2] Jan Łukasiewicz. ***Aristotle's Syllogistic From the Standpoint of Modern Logic***, Clarendon Press, Oxford, 1951.

[3] Tahsin Oner, Ibrahim Senturk, *The Sheffer Stroke Operation Reducts of Basic Algebras*, ***Open Mathematics***, vol.15, no.1, pp.926–935, 2017.

[4] Ruggero Pagnan, *A Diagrammatic Calculus of Syllogisms*, ***journal of logic language and information***, no.21, pp.347–364, 2012.

[5] A. E. Kulinkovich, *Algorithmization of Resoning in Solving Geological Problems*, ***Proceedings of the Methodology of Geographical Sciences*** Naukova Dumka, 1979, pp. 145–161.

[6] Ibrahim Senturk, Tahsin Oner, *An Algebraic Analysis of Categorical Syllogisms by Using Carroll's Diagrams*, ***Filomat***(Accepted).

[7] Ibrahim Senturk, Tahsin Oner, *A Construction of Heyting Algebra on Categorical Syllogisms*, ***Matematichki Bilten***, vol.40, no.4, pp.5–12, 2016.

Mathematics Department, Faculfy of Science, Ege University, 35100, Bornova, Izmir, Turkey

`ibrahim.senturk@ege.edu.tr`

Paul Shafer

Comparing the degrees of enumerability and the closed Medvedev degrees

joint work with Andrea Sorbi

For sets $\mathcal{A}, \mathcal{B} \subseteq \omega^\omega$, recall that $\mathcal{A}$ *Medvedev reduces* to $\mathcal{B}$ if there is a Turing functional Φ such that $\Phi(f) \in \mathcal{A}$ whenever $f \in \mathcal{B}$. The degree structure induced by Medvedev reducibility is called the *Medvedev degrees*.

Both the Turing degrees and the enumeration degrees embed into the Medvedev degrees: map the Turing degree of $f \in \omega^\omega$ to the Medvedev degree of $\{f\}$, and map the enumeration degree of (a nonempty) $A \in 2^\omega$ to the Medvedev degree of $\{g : \mathrm{ran}(g) = A\}$. The embedding of the Turing degrees into the Medvedev degrees is particularly nice. Every Turing degree is mapped to the Medvedev degree of a closed set (in particular, a singleton), and the range of the embedding is definable (a theorem of Dyment and Medvedev). On the other hand, little is known about the embedding of the enumeration degrees in the Medvedev degrees. For example, whether or not the range of the embedding is definable is a longstanding open question of Rogers.

Call a Medvedev degree *closed* if it is the degree of a closed subset of ω^ω, and call a Medvedev degree a *degree of enumerability* if it is in the range of the embedding of the enumeration degrees into the Medvedev degrees. We explore the distribution of the degrees of enumerability with respect to the closed degrees and find that many situations occur. There are nonzero closed degrees that do not bound nonzero degrees of enumerability, there are nonzero degrees of enumerability that do not bound nonzero closed degrees, and there are degrees that are nontrivially both degrees of enumerability and closed degrees. We also show that the compact degrees of enumerability exactly correspond to the cototal enumeration degrees.

School of Mathematics, University of Leeds, Leeds LS2 9JT, United Kingdom
p.e.shafer@leeds.ac.uk

Assaf Shani

Borel equivalence relations and symmetric models

We develop a correspondence between the study of Borel equivalence relations induced by closed subgroups of S_∞, and the study of symmetric models of set theory without choice, and apply it to solve questions of [HKL98]. In [HKL98], the possible values of *potential complexity* of Borel equivalence relations which are induced by actions of closed subgroups of S_∞ are completely classified. To that end, they refine the Friedman-Stanley hierarchy $\cong_\alpha$, $\alpha < \omega_1$, by defining equivalence relations $\cong^*_{\lambda+1}$, $2 \leq \lambda < \omega_1$. E.g., $\cong^*_n$ has potential complexity $D(\Pi^0_n)$. Moreover, they define equivalence relations $\cong^*_{\lambda+1,0} \leq_B \cong^*_{\lambda+1}$, and show that they correspond to actions by "well-behaved" closed subgroups of S_∞. That is, if $E \leq_B \cong^*_{\lambda+1}$ is induced by a Borel G-action of a closed subgroup G of S_∞ *which admits an invariant compatible metric*, then $E \leq_B \cong^*_{\lambda+1,0}$. Furthermore, they prove that for any countable ordinal α, $\cong^*_{\alpha+3,0} <_B \cong^*_{\alpha+3}$. They ask whether the remaining reductions are also strict, and conjecture that they are. We confirm this, and focus on the minimal open cases:

Theorem 1. $\cong^*_{\omega+1,0} <_B \cong^*_{\omega+1}$ *and* $\cong^*_{\omega+2,0} <_B \cong^*_{\omega+2}$.

The proof goes through studying symmetric models generated by generic invariants of these equivalence relations. To make the connection with Borel reducibility we use tools developed by Zapletal. We use models constructed in [Mon73], separating the "*generalized Kinna-Wagner principles*", KW^n, which state that every set can be injected into the n'th-power set of an ordinal. Towards proving Theorem 1, we show the consistency of $\mathrm{KW}^{\omega+1} \wedge \neg\mathrm{KW}^{\omega}$, answering a question of [Kar1?].

References

[HKL98]G. Hjorth, A. S. Kechris and A. Louveau, *Borel equivalence relations induced by actions of the symmetric group*, ***Ann. Pure Appl. Logic***, 92 (1998), 63-112.

[Kar1?]A. Karagila, *Iterating Symmetric Extensions*, ***ArXiv 1606.06718***.

[Mon73]G. P. Monro, *Models of ZF with the same sets of sets of ordinals*, ***Fund. Math.***, 80 (1973), no. 2, 105-110.

Department of Mathematics, University of California Los Angeles, CA 90095-1555, USA
assafshani@ucla.edu

Andrei Sipoş

Quantitative results on the method of averaged projections

A recent direction of research in convex optimization has been the extension of classical results that hold in normed spaces to various nonlinear analogues, e.g. the class of CAT(0) spaces. One of such extensions refers to the class of *firmly nonexpansive* mappings (that play a central role in convex optimization, as they encompass a large number of concrete and useful cases such as proximal mappings or resolvents), whose nonlinear generalization was introduced in [1]. A slightly larger class of mappings (which coincides with the former in the context of Hilbert spaces) consists of those that satisfy *property* (P_2), introduced in [2], where the fact that the composition $T := T_2 \circ T_1$ of two such mappings with $Fix(T) \neq \emptyset$ is asymptotically regular.

The primary application of this kind of algorithm is the *alternating projections method*, where the two mappings are the projection operators on two closed, convex, nonempty subsets A and B of the space X. The goal is, then, to find *best approximation pairs* corresponding to those sets, i.e. pairs $(a, b) \in A \times B$ such that $d(a, b) = d(A, B)$, or, at least, points that approximate such pairs to any prior degree.

A related method, the *method of averaged projections*, replaces the composition of projections from above with a weighted average of them, i.e. a convex combination. A classical way of proving the efficacy of this method in the usual Euclidean space (and recently also in Hilbert spaces [3, Section 5]) is by reducing it to the first one through the replacing of the combined mapping with a concatenated transformation in a product space with the averaged metric, followed by a projection on the diagonal. Our goal is to extend that line of argument to self-mappings satisfying property (P_2) – in particular, to firmly nonexpansive self-mappings – in the setting of CAT(0) spaces.

In addition, we will also establish effective versions of such asymptotic results, in the sense of proof mining. Proof mining is an applied subfield of mathematical logic, developed primarily by U. Kohlenbach and his collaborators ([4] is the standard introduction, while more recent surveys are [5, 6]), that aims to provide quantitative information (witnesses and bounds) for numerical entities which are shown to exist by proofs which cannot be necessarily said to be fully constructive. In nonlinear analysis, which has been a primary focus for such work, the relevant information is usually found within convergence statements, where the problem is to find an explicit formula for the N_ε such that for any $\varepsilon > 0$, the elements of the sequence of index greater than N_ε are ε-close to the limit. The result above is an instance of this, as asymptotic regularity is clearly a statement of convergence, and it has indeed been analyzed from this viewpoint in [7].

The results presented in this talk may be found in [8].

References

[1] D. Ariza-Ruiz, L. Leuştean, G. López-Acedo, *Firmly nonexpansive mappings in classes of geodesic spaces*, ***Transactions of the American Mathematical Society***, vol. 366 (2014), pp. 4299–4322.

[2] D. Ariza-Ruiz, G. López-Acedo, A. Nicolae, *The asymptotic behavior of the composition of firmly nonexpansive mappings*, ***Journal of Optimization Theory and Applications***, vol. 167 (2015), pp. 409–429.

[3] H. Bauschke, V. Martin-Marquez, S. Moffat, X. Wang, *Compositions and convex combinations of asymptotically regular firmly nonexpansive mappings are also asymptotically regular*, ***Fixed Point Theory and Applications***, vol. 2012 (2012), no. 53.

[4] U. Kohlenbach, ***Applied proof theory: Proof interpretations and their use in mathematics***, Springer Monographs in Mathematics, Springer-Verlag, 2008.

[5] ——— *Recent progress in proof mining in nonlinear analysis*, ***IFCoLog Journal of Logics and their Applications***, vol. 10 (2017), no. 4, pp. 3357–3406.

[6] ——— *Proof-theoretic methods in nonlinear analysis*, Draft, to appear in ***Proceedings of the ICM2018***.

[7] U. Kohlenbach, G. López-Acedo, A. Nicolae, *Quantitative asymptotic regularity for the composition of two mappings*, ***Optimization***, vol. 66 (2017), pp. 1291–1299.

[8] A. Sipoş, *The asymptotic behaviour of convex combinations of firmly nonexpansive mappings*, arXiv:1802.08476 [math.OC], 2018.

Department of Mathematics, Technische Universität Darmstadt, Schlossgartenstrasse 7, 64289 Darmstadt, Germany
Simion Stoilow Institute of Mathematics of the Romanian Academy, Calea Griviţei 21, 010702 Bucharest, Romania
sipos@mathematik.tu-darmstadt.de

William Stafford

The untyped λ-calculus as a foundation for natural language semantics: The problem of computational intractability

Natural language appears to contain hyperintensional settings, but it has proven difficult to formalise hyperintensionality using Montague's approach of treating intensions as functions from worlds to assignments. Fox and Lappin (2005) have proposed Property Theory with Curry Types (PTCT) as an alternative foundation for natural language. They claim that PTCT formalises hyperintensionality and polysemy but is a computationally weaker system than the traditional Montague grammars. In this paper we dispute their claim to have generated a computationally weaker system. Firstly, their argument is based on the claim that Montague grammars are second-order systems and so stronger than PTCT, which is a first order system. Second-order systems are only stronger than first-order systems if we use the full semantics. This means that Montague grammars are of equivalent strength to PTCT if they use a weaker semantics. Secondly, PTCT uses the untyped λ-calculus while Montague grammars uses the typed λ-calculi. The untyped calculus is Turing complete, while the typed systems are not. From this point of view, we see that PTCT might in fact be a stronger formal system than Montague grammars.

References

[1] Fox, Chris and Lappin, Shalom , ***Foundations of Intensional Semantics***, Blackwell Publishing, 2005.

University of California, Irvine
stafforw@uci.edu

Gergely Székely

Frames and coordinate systems in the formalization of Einstein's special principle of relativity

joint work with Judit X. Madarász and Mike Stannett

In the literature there are several informal treatments and discussions of Einstein's special principle of relativity (SPR) and its consequences. The main problem with informal approaches in physics (and science in general) is that they often lead to confusions and misunderstandings.

Of course, formalization in itself does not save us from bumping into apparent contradictions. For example Rindler, in his book [3, p.40] referring to Dixon, claims that "the principle of relativity is equivalent to the isotropy (of space) and the homogeneity (of space and time)". Contrary to this claim, the construction proving Theorem 2 in paper [1] gives an anisotropic extension of the standard model of special relativity which still satisfies SPR These two claims appear to be a direct contradictions of each other. Of course, the contradiction is only apparent since something else is meant by 'the principle of relativity' in [1] and [3]. Even the mathematical frameworks of the two formulations are different. So there are inequivalent formulations of SPR in the literature. Therefore, it seems so natural to ask: which formulation is the 'true one'?

The 'original' SPR is just an informal idea that goes back at least to Galileo's famous Dialogo. So it is not surprising that it has several different formalizations since an idea can clearly be formulated in several different ways based on the many choices one has to make when turning an idea into a formal statement. Therefore, the right question is not which formulation is the 'true one', but how the different formulations are related to one another logically.

In [2], we have already compared three different formalizations of SPR within a mathematical logic based axiomatic framework developed by the Andréka–Németi group and investigated various auxiliary assumptions that make these three formalizations equivalent. Now we are going to use the same framework and Rindler's distinctions between inertial frames and inertial coordinate systems to investigate the logical connection between the two versions of the principle from [1] and [3]. We will see that SPR in [3] is understood for coordinate systems and the construction in [1] satisfies SPR understood for reference frames.

Based on Galileo's ship argument, we will also argue that the original intuition behind SPR is in some sense better reflected if we formulate it for reference frames only and hence does not imply isotropy.

References

[1] H. Andréka, J. X. Madarász, I. Németi, M. Stannett and G. Székely, *Faster than light motion does not imply time travel*, ***Classical and Quantum Gravity***, vol. 31 (2014), no. 9, Paper 095005, 11 pp.

[2] M. Stannett, J. X. Madarász, and G. Székely, *Three Different Formalisations of Einstein's Relativity Principle*, ***The Review of Symbolic Logic***, vol. 10 (2017), no. 3, pp. 530–548.

[3] W. Rindler, ***Relativity: Special, General, and Cosmological***, Oxford University Press, Oxford, 2001.

Alfréd Rényi Institute of Mathematics, Hungarian Academy of Sciences
szekely.gergely@renyi.mta.hu

Ramanathan S. Thinniyam

Decidability in the substructure ordering of finite graphs

joint work with Anantha Padmanabha

Let $\mathcal{G}$ be the set of isomorphism types of finite, undirected, simple graphs and $\leq$ the substructure ordering (also called induced subgraph order) on $\mathcal{G}$. It was shown in [1] that every computable relation over graphs is definable in first order logic over the structure $(\mathcal{G}, \leq, P_3)$, where P_3 is a constant symbol interpreted as the path on three vertices. Thus the first order theory $Th(\mathcal{G}, \leq, P_3)$ is extremely rich and highly undecidable. We study the boundary between decidability and undecidability using syntactic restrictions.

We show that the purely existential theory $Th_{\exists^*}(\mathcal{G}, \leq)$ is decidable, and in fact, $NP-$complete. But on addition of a constant symbol for every graph to the vocabulary, the existential theory $Th_{\exists^*}(\mathcal{G}, \leq, \mathcal{C})$ becomes undecidable.

In the case of finite variable restrictions, the single variable fragment $Th_{FO^1}(\mathcal{G}, \leq, \mathcal{C})$ is decidable while the three-variable fragment $Th_{FO^3}(\mathcal{G}, \leq, \mathcal{C})$ is undecidable. The decidability of the two-variable fragment of the induced subgraph order with constants is open; and is related to some interesting combinatorial questions about graphs. The analogous combinatorial questions in the case of the subword order studied in [2] can be seen a special case of the questions for graphs.

References

[1] R. S. Thinniyam, *Defining recursive predicates in graph orders*, ***Logical Methods in Computer Science***, Accepted for publication. https://arxiv.org/pdf/1709.03060.pdf

[2] P. Karandikar, Ph. Schnoebelen, *The height of piecewise-testable languages with applications in logical complexity*, ***CSL'16, Leibniz International Proceedings in Informatics*** (Computer Science Logic 2016, Marseille, France), (Laurent Regnier and Jean-Marc Talbot, editors), vol. 62, Leibniz-Zentrum für Informatik, 2016, pp. 37:1-37:22.

The Institute of Mathematical Sciences, 4th Cross Street, CIT campus, Taramani, Chennai 600113, India
thinniyam@imsc.res.in

Carlo Toffalori

The torsion free part of the Ziegler spectrum of orders over Dedekind domains

We introduce and illustrate recent results obtained with Lorna Gregory and Sonia L'Innocente on the model theory of modules over orders over Dedekind domains, in particular on the torsion free part of their Ziegler spectrum. These results follow some suggestions of Gena Puninski and enlarge a past analysis with Mike Prest and Annalisa Marcja over group rings. Here orders are meant as certain widely considered finitely generated ring extensions of Dedekind domains.

Division of Mathematics, School of Science and Technology, University of Camerino, Via Madonna delle Carceri 9, Camerino, Italy
carlo.toffalori@unicam.it

Jouko Väänänen

An extension of a theorem of Zermelo

We show that if $(M, \in_1, \in_2)$ satisfies the first order Zermelo-Fraenkel axioms of set theory when the membership-relation is $\in_1$ and also when the membership-relation is $\in_2$, and in both cases the formulas are allowed to contain both $\in_1$ and $\in_2$, then $(M, \in_1) \cong (M, \in_2)$, and the isomorphism is definable in $(M, \in_1, \in_2)$. This extends a theorem of Zermelo [2] from 1930. A similar result holds for first order Peano arithmetic, extending the categoricity result of Dedekind [1] of second order Peano arithmetic. The proof of this is similar, but somewhat easier.

References

[1] R. Dedekind. *Was sind und was sollen die Zahlen?* Braunschweig. Vieweg *and* Sohn, 1888.

[2] E. Zermelo. Über Grenzzahlen und Mengenbereiche. Neue Untersuchungen über die Grundlagen der Mengenlehre. *Fundam. Math.*, 16:29–47, 1930.

University of Helsinki and University of Amsterdam
jouko.vaananen@helsinki.fi

Diego Valota

Computing Spectra of Many-Valued Logics, via Dualities

WNM is a many-valued logic that lies in the MTL hierarchy [4]. Several t-norms based logics, such as Gödel, NM, NMG, and RDP logics, are schematic extensions of WNM. The aim of this talk is to show how to solve *spectra problems* for these logics, by exploiting spectral dualities of their corresponding varieties of algebras. In particular from the subforests representation theorem of finite Gödel algebras, we obtain formulas to compute free finitely generated Gödel algebras (*free spectra*) [1], and non-isomorphic finite Gödel algebras (*fine spectra*) [5]. As a consequence we obtain the set of cardinalities of Gödel algebras (*spectra*).

Using results in [3, 6, 2] we show how to enrich the category of finite forests and open maps, to generalize the above procedures for other logics in WNM hierarchy.

References

[1] S. Aguzzoli, S. Bova, and B. Gerla, *Free algebras and functional representation for fuzzy logics*, ***Handbook of Mathematical Fuzzy Logic. Vol.2*** (P. Cintula and P. Hájek, and C. Noguera, editors), College Publications, 2011, pp. 713–791.

[2] S. Aguzzoli, S. Bova, and D. Valota, *Free weak nilpotent minimum algebras*, ***Soft Computing***, vol. 21 (2017), no. 1, pp. 79–95.

[3] S. Bova and D. Valota, *Finitely Generated RDP-Algebras: Spectral Duality, Finite Coproducts and Logical Properties*, ***Journal of Logic and Computation***, vol. 22 (2012), no. 3, pp. 417–450.

[4] F. Esteva and L. Godo, *Monoidal t-Norm Based Logic: Towards a Logic for Left-Continuous t-Norms*, ***Fuzzy Sets and Systems***, 124(3):271–288, 2001. vol. 124 (2001), pp. 271–288.

[5] D. Valota, *Representations for Logics and Algebras related to Revised Drastic Product t-norm*, ***Submitted***

[6] D. Valota, *Spectra of Gödel Algebras*, ***Submitted***

Dipartmento di Informatica, Università di Milano, Via Comelico 39, I-20135 Milano, Italy
valota@di.unimi.it

Caterina Viola

Piecewise linear valued constraint satisfaction problems

joint work with Manuel Bodirsky and Marcello Mamino

Valued constraint satisfaction problems (VCSPs) are a class of combinatorial optimisation problems. Recently, the computational complexity of all VCSPs for finite sets of cost functions over finite domains has been classified completely. Many natural optimisation problems, however, cannot be formulated as VCSPs over a finite domain. One example is the famous linear programming problem, where the task is to optimise a linear function subject to linear inequalities. This can be modelled as a VCSP over $\mathbb{Q}$, the set of rational numbers, by allowing unary linear cost functions and cost functions of higher arity to express the crisp (i.e. hard) linear inequalities. I will present our work that initiates the systematic investigation of infinite-domain VCSPs by studying the complexity of VCSPs for classes of piecewise linear cost functions, i.e., cost functions admitting a first-order definition on $(\mathbb{Q}; +, <, 1)$.

Technische Universität Dresden
caterina.viola@tu-dresden.de

Farn Wang

Statistical Model-Checking Uncertain Responses written in LTL with Confidence

joint work with Wen-Hong Chiang

Model-checking [1] has been an important technology that aims to bridge the logic community and the industry. However, the assumption of rigid and complete models of the model-checking technology has been a major incompatibility block to the industry projects. In practice, there is usually no complete and precise model at all. Moreover, most specification properties cannot be simplistically evaluated as 'true' (for satisfaction) or 'false' (for failure). This is particularly true in the case of evaluating system responses. For example, if we try to download a music file from a server via the internet. Usually the users can tolerate a few downloading failures. To avoid frustration of the users, the developers may also want to lift the download success ratio to $p = 95\%$ if the download request immediately follows an unfulfilled request. This can be specified with the following *Linear Temporal Logic* (*LTL*) property [3]:

$$\Box((\mathit{downloadFail} \wedge \bigcirc(\mathit{downloadOrCancel} \wedge \bigcirc \mathit{download})) \rightarrow \bigcirc\bigcirc\bigcirc \mathit{downloadSucc})$$

But clearly the model-checking technology cannot aid in answering how many consecutive failures can be compiled to a confident test verdict of a server defect.

In this work, we adopt the LTL statistical model-checking [4] which extends the LTL model-checking problem to the following.

Definition 1. Model Set Evaluation Problem with Chernoff Confidence (MSEPCC) Given a set S of traces (for a system with uncertain responses due to environment interference) and an LTL formula ϕ, what is the Chernoff Bound of probability that S is an evidence that the system implements ϕ ? ⊣

We have implemented a tool, *System Response Tester with Confidence* (*SRTC*), that utilizes the classic concept of Chernoff bounds [2] for Bernoulli variables and sample sizes to calculate a bound on probability that a set of randomized traces is an evidence of system failure to achieve a response success ratio. There are the following technical issues in collecting trace sets for evaluating Chernoff bounds.

- **Efficiency:** The generation of traces cannot be too random for our purpose to trigger the precondition for the target response property.

- **Randomness:** The Chernoff bounds are based on the assumption that the samples are randomly collected. We propose to randomize the lengths of the test cases. Intuitively, the lengths of test sessions should constitute a Poisson distribution.

SRTC can efficiently generate trigger sequences for a given response property with lengths of the sequences roughly constitute a Poisson distribution. We experimented with several Android apps from Google play. Experiment data shows that SRTC could be handy in evaluating issues related to response properties expressed in LTL with the confidence values derived from a Chernoff bound.

References

[1] E. M. Clarke, E.A. Emerson, *Design and Synthesis of Synchronization Skeletons using Branching-Time Temporal Logic*, ***Workshop on Logic of Programs***, LNCS 131, Springer-Verlag, 1981.

[2] H. Chernoff, *Asymptotic efficiency for tests based on the sum of observations*, ***Annals of Mathematical Statistics***, vol. 23, pp. 493-507, 1952.

[3] A. Pnueli, *The Temporal Logic of Programs*, ***18th annual IEEE-CS Symposium on Foundations of Computer Science***, pp. 45-57, 1977.

[4] , H.L.S. Younes, R.G. Simmons, *Statistical probabilistic model checking wiith a focus on time-bounded properties*, ***Information and Computation***, 2006, 204(9), pp.1368âĂŞ1409, Elsevier.

Depart. of Electrical Engineering, National Taiwan University
farn@ntu.edu.tw

Wei Wang

Some variants of weak pigeonholes principle and WWKL

joint work with David Bélanger, Chitat Chong, Tin Lok Wong, and Yue Yang

Recently Avigad et al. [Avigad.Dean.ea:2012] introduced a variant of WWKL, $2-\mathsf{WWKL}$, which says that if a Δ^0_2 binary tree T and a positive ration satisfy $\forall n(|T \cap 2^n| > \delta 2^n)$ then T has an infinite path. They proved that $2-\mathsf{WWKL}$ implies $B\Sigma^0_2$ over RCA_0. Combining this with a theorem of Conidis and Slaman [Conidis.Slaman:2013], we know that the first order theory of $2-\mathsf{WWKL}$ is axiomatized by $B\Sigma^0_2$. Obviously, $2-\mathsf{WWKL}$ implies $2-\mathsf{RAN}$, the existence of a 2-random. Slaman proved that $\mathsf{RCA}_0 + 2 - \mathsf{RAN} \nvdash B\Sigma_2$. But the first order theory of $\mathsf{RCA}_0 + 2 - \mathsf{RAN}$ remains unknown.

Here we introduce an interpolation between $2 - \mathsf{WWKL}$ and $2 - \mathsf{RAN}$, denoted by $2 - \mathsf{WWKL}(\geq 2^{-1})$, which says that if a Δ^0_2 binary tree T satisfies $\forall n(|T \cap 2^n| \geq 2^{n-1})$ then T has an infinite path. It turns out that the first order theory of $\mathsf{RCA}_0 + 2 - \mathsf{WWKL}(\geq 2^{-1})$ has interesting connections with $\Sigma^0_2 - \mathsf{WPH}$ (there exists no Σ^0_2 injection from any $2a$ to a) where WPH stands for Weak Pigeonholes Principle, and the cardinal scheme (there exists no first order definable injection from $\mathbb{N}$ to any a). For example, the first order theory of $2 - \mathsf{WWKL}(\geq 2^{-1})$ is axiomatized by $\Sigma^0_2 - \mathsf{WPH}$ which lies strictly between $B\Sigma^0_2$ and the cardinal scheme for Σ^0_2 injections.

Department of Philosophy and Institute of Logic and Cognition, Sun Yat-sen University, Guangzhou 510275, P. R. China
wwang.cn@gmail.com

Bartosz Wcisło

Non-speed-up results for purely compositional truth predicates

joint work with Mateusz Łełyk

In the talk we discuss some concrete instantiations of the following general problem: which conservative extensions of a given axiomatic theory Th enable much shorter proofs of the theorems of Th? Next definition clarifies the intuitions behind this question:

Definition 1 (Speed-up). Let Th_1 and Th_2 be two theories and Φ a set of functions $\mathbb{N} \to \mathbb{N}$. We shall say that Th_2 has a *super-Φ speed-up over* Th_1 if there exists an infinite sequence of formulae $\phi_0, \phi_1, \dots$, provable in both Th_1 and Th_2 such that for every function $f \in \Phi$, there exists $k \in \mathbb{N}$ such that for every $n \geq k$ we have

$$\parallel \phi_n \parallel_{\mathrm{Th}_1} > f(\parallel \phi_n \parallel_{\mathrm{Th}_2}),$$

where $\parallel \phi_n \parallel_{\mathrm{Th}}$ denotes the length of the binary code of the shortest proof of ϕ in Th.

Note that when investigating lengths of proofs we focus on the number of symbols used in the derivation and not just on the number of proof-lines. One can show, for example, that ACA_0 has a super-elementary speed-up over PA, while being conservative over it. The same holds for the pair GB and ZFC. These results together with the above definition are presented in [6].

In the talk we discuss the case of three *axiomatic theories of truth*, i.e. axiomatic theories extending Peano Arithmetic (PA) formulated in the language with symbols for arithmetical operations and one fresh unary predicate $T(x)$, thought of as *the truth predicate*. The distinctive feature of the theories we consider is that all of them are purely compositional: the unique new axioms added to PA state how the truth of compound sentences depend on the truth of its immediate subformulae. In particular we do not add to any of the theories any induction axioms for formulae with the truth predicate.

The core construction concerns the theory of basic compositional truth for the language of arithmetic, known as CT^-.[1] Axioms for the predicate T in this theory are the natural arithmetization of Tarski's inductive definition of truth (stated in the extended language but *only for the language of arithmetic*). CT^- is well-known to be conservative over PA and three essentially different proofs demonstrating this can be found in [3] (KKL Theorem), [5] and [4].

In our presentation we sketch the proof that CT^- does not have a super-polynomial speed-up over PA. The proof idea has been given by Ali Enayat and it consists in a neat arithmetization of the model-theoretic conservativity proof of CT^- over PA, given in [4]. More concretely, it can be shown that there exists a *feasible interpretation* of CT^- in PA, i.e. one in which the translation of every axiom is provable in PA with a proof of length polynomial in the length of the axiom.

Next, we discuss the application of this method to two untyped axiomatic theories of truth: KF^- and FS^-.[2] Both of them contain only compositional axioms for the truth predicate, but for *the extended language* (and not only the language of arithmetic, as in CT^-). What makes them consistent is the fact that neither of them is *fully compositional*: KF^- allows for formulae being neither true nor false or both true and false while FS^- does not admit the reiteration axiom $\forall\phi \ \big(T(T(\phi)) \equiv T(\phi)\big)$.

In both cases we show that the proof of conservativity of the respective theory over PA (known from the literature) gives rise to a feasible interpretation of this theory in PA.

[1] Called also $\mathrm{CT}\!\restriction$ in [2] and CT[PA] in [5]

[2] Defined as $\mathrm{KF}\restriction$ and $\mathrm{FS}\restriction$ in [2]

References

[1] Hájek P., Pudlák P., ***Metamathematics of First-Order Arithmetic***, Springer-Verlag, 1998.

[2] Halbach V., ***Axiomatic Theories of Truth***, Cambridge University Press, 2014.

[3] Kaye R., ***Models of Peano Arithmetic***, Oxford University Press, 1991.

[4] Enayat A., Visser A., *New Construction of Satisfaction Classes*, ***Unifying the Philosophy of Truth*** (Achourioti T., Galinon H., Martínez Fernández J., and Fujimoto K.), Springer-Verlag, 2015, pp. 321–335.

[5] Leigh G., *"Conservativity of Theories of Compositional Truth via Cut Elimination*, ***Journal of Symbolic Logic***, vol. 80 (3), pp. 845–865.

[6] Pudlak P., *The lengths of proofs*, ***Handbook of Proof Theory*** (S.R. Buss), Elsevier, 1998, pp. 547–637.

Institute of Mathematics, University of Warsaw
bar.wcislo@gmail.com

Michał Wrocławski

Computability of functions and sets in various representations of natural numbers

joint work with Dariusz Kalociński

To perform computations on natural numbers (or any other mathematical objects) we need to represent them somehow. By a representation of natural numbers we mean a pair (S, σ), where S is a decidable set of numerals (finite inscriptions) over a finite alphabet and σ is a surjective function which assigns to each numeral a natural number. Unlike Shapiro in [1], we assume that this function is possibly non-injective.

A function is computable in a certain representation if there exists an algorithm which for every numeral representing an argument, returns a numeral representing the value of the function. If we assume that a certain function is computable in a given representation, we would like to know if it follows that certain other functions are computable in it too. We show that - when considering functions like addition, multiplication or exponentiation - such an implication usually does not hold.

However, the situation changes when there is an algorithm which can decide whether two given numerals represent the same number. Then the computability of addition or exponentiation becomes a fairly strong assumption - it allows us to draw some conclusions about computability of other functions.

Another important issue is computability of characteristic functions of different sets or relations. It turns out that it is completely independent from computability of numerical functions, i.e. for any countable F - set of numerical functions and any $C \subseteq \mathbb{N}, C \neq \emptyset, C \neq \mathbb{N}$ there exists a representation in which all functions from F are computable but the characteristic function of C is not computable.

References

[1] Stewart Shapiro, *Acceptable notation*, ***Notre Dame Journal of Formal Logic***, vol. 23 (1982).

Institute of Philosophy, University of Warsaw, ul. Krakowskie Przedmieście 3, 00-927 Warsaw, Poland
michalwro@wp.pl

Kaibo Xie

Reasoning about causal belief

Pearl [2] provides an elegant and descriptively strong account of causal inference using Causal Models which represent causal dependencies between variables using structural equations. Halpern [2] develops a logic for causal reasoning based on the Causal Models in which truth conditions of counterfactuals are defined in terms of the consequences of changes in values of variables, as described by structural equations.

Halpern's logic is able to express counterfactual statements; however, it cannot express beliefs in counterfactuals. In order to make up this part, our proposal extends the model with a plausibility ordering and extends the language for causal reasoning with a belief operator as well.

Halpern (2016) has introduced a way of extending Causal Models with an ordering over possible worlds based on plausibility measures; yet, the ordering in Halpern's approach stands for normality instead of beliefs. In our model, the plausibility ordering represents the agent's beliefs using the techniques provided by Baltag and Smets [1]: beliefs can be defined in terms of probabilistic distribution over possible worlds, which can be qualitatively translated into a plausibility ordering over them. Thus, beliefs boil down to a plausibility ordering.

By extending models with plausibility orderings, which qualitatively represent probability, we can define the truth condition of "believing a counterfactual" within the extended model. This allows us to extend the object language with belief operators, so that it is possible to develop a logic which not only derives causal inference but also derives what can be inferred from causal beliefs.

This extension is useful in many applications, as in distinguishing different readings of counterfactuals, explaining the inference from "knowing the value", and so on.

References

[1] A. Baltag and S. Smets, *Probabilistic dynamic belief revision*, ***Synthese***, 165(2):179-202, 2008

[2] J. Y. Halpern, ***Actual Causality***, MIT Press, 2016.

[3] J. Pearl, ***Causality: models, reasoning and inference***, IIE Transactions, 2002.

Institute for Logic, Language and Computation, University of Amsterdam, 1090 GE Amsterdam, The Netherlands
K.Xie@uva.nl

Aibat Yeshkeyev

Hereditary β-cosemantic Jonsson theories

It is well known that the perfect Jonsson theory can be studied using of the first-order properties of the center of this theory and its semantic model, since the center of the Jonsson theory is the model companion of it. Imperfect Jonsson theories at the moment have not been studied. For example, a bright example of this fact is the theory of all groups. We know that this theory is Jonsson, but does not have a model companion, and the structure of its semantic model is unknown to us. In this thesis, we will consider a special subclass of Jonsson theories, namely, hereditary β-cosemantic Jonsson theories. The need to introduce such a class was due to the following questions: 1) to find a reasonable approach to describing imperfect Jonsson theories; 2) finding the conditions for preserving the theory's consistency with some enrichment of the language; 3) finding conditions for preserving the definability of a type with the appropriate stability obtained as a result of enriching the language. The proposed method for solving the first problem is to use the central type and forcing companion in some permissible enrichment of the language of the Jonsson theory under consideration. The second and third tasks appear automatically after the proposed method to solve the first problem. All three problems are independent in themselves and they have connections with known questions for complete theories. For example, in problem 2 there is a question about the existence of an amalgam and joint embedding property. In connection with Problem 3, well known are the results of Mustafin T.G. [1] and Palyutin E.A. [2] about new types of stability for complete theories. In the works of B. Poizat [3], E. Bouscaren [4] considered the problem of completeness of elementary pairs. It is clear that the concepts of heredity and β-cosemanticity will be a refinement of these problems in the framework of studying, generally speaking, incomplete Jonsson theories.

We introduce the following definitions necessary for the above purposes.

Definition 1. An enrichment $\overline{\top}$ of the Jonsson theory T is said to be permissible if any type in this enrichment is definable in the framework of $\overline{\top}$-stability.

Definition 2. The Jonsson theory is said to be hereditary, if in any of its permissible enrichment any expansion of it in this enrichment will be Jonsson theory.

Definition 3. The Jonsson theory T is called (α, β_i)-cosemantic, where $i \in \overline{1, \alpha}$, if the quotient of the Jonsson spectrum [5] of any model of the theory T by cosemanticness has α equivalence classes, each of which has the power β_i, respectively.

If $\alpha = 1$, then we omit the first index and say that the Jonsson theory T β-cosemantic.

References

[1] Mustafin T.G., *New concepts of the stability of theories*, ***Proceedings of the Soviet-French Colloquium on Model Theory***, Karaganda, 1990, pp. 112–125.

[2] Palyutin E.A., *E^*-stable theories*, ***Algebra and Logic***, vol. 41 (2003), no. 2, pp. 194–210.

[3] Poizat B., *Paires de structures stables*, ***Journal of Symbolic Logic***, vol. 48 (1983), no. 2, pp. 239–249.

[4] Bouscaren E., *Elementary pairs of models*, ***Annals of Pure and Applied Logic***, vol. 45 (1989), pp. 129–137.

[5] Yeshkeyev A.R., Ulbrikht O.I., *JSp-cosemanticness and JSB property of Abelian groups*, ***Siberian electronic mathematical reports***, vol. 13 (2016), pp. 861–874.

Faculty of Mathematics and Information Technologies, Karaganda State University, University str., 28, building 2, Kazakhstan
aibat.kz@gmail.com

Léo Zaradzki

The chair might not have barked

In the last years, researchers in natural language semantics have been developing frameworks based on many-sorted logic: on the lexical side it helps to capture the various facets of a given word, while on the compositional side it enables to block intuitively *ill-typed* sentences like "The chair barked" (this precisely reflects the reason why type theories were primarily introduced in mathematics and in computer science). However, there is no consensus about the status to be given to negations of such infelicitous statements, like "The chair did not bark". Some say they are infelicitous, while others say they are well-formed (and true) propositions. The same contrast can be made about ontological statements like "This is not a fish", which is good when pointing to a dolphin, but not to a chair. In this talk, we shall discuss how such negative statements may be accounted for in extensions of Montague semantics to richer type-theoretical frameworks. More generally it can shed a new light on the debate for knowing what the primitive types should be.

References

[1] Stergios Chatzikyriakidis and Zhaohui Luo, editors. ***Modern Perspectives in Type Theoretical Semantics***, Springer, Berlin/Heidelberg, Germany, 2017.

[2] Nicholas Asher, ***Lexical meaning in context – a web of words***, Cambridge University Press, 2011.

[3] Christian Retoré, *The Montagovian Generative Lexicon ΛTy_n: a Type Theoretical Framework for Natural Language Semantics*, ***Leibniz International Proceedings in Informatics*** (Dagstuhl, Germany), (Ralph Matthes and Aleksy Schubert, editors), vol. 26, Schloss Dagstuhl–Leibniz Zentrum für Informatik, 2014, pp. 202–229.

LLF and CRI, Université Paris Diderot, 5 rue Thomas Mann 75013 Paris, France
leozaradzki@gmail.com

Maxim Zubkov

On categoricity of linear orders

A computable algebraic structure is called Δ^0_α-categorical if for any two computable copy of it there exists a Δ^0_α-isomorphism between of them. A computable algebraic structure is called relatively Δ^0_α-categorical if for any two x-computable copy of it there exists a Δ^x_α-isomorphism between of them. S.S. Goncharov and V.D. Dzgoev [1] and, independently, J.B. Remmel [2] gave a characterization of computable categorical linear orders. They proved that a computable linear order is computably categorical iff it has finitely many successors. C. McCoy [3] gave a characterization of relatively Δ^0_2 categorical linear orders. He proved, that if a linear order $\mathcal{L}$ has a computable copy with a computable successor relation, and computable left and right limit points then $\mathcal{L}$ is Δ^0_2 categorical iff it is relatively Δ^0_2 categorical. C. Ash [5] find levels of categoricity of constructive ordinal, and N. Bazhenov [6] find degrees of categoricity of constructive ordinals. Namely, C. Ash proved that if an ordinal α such that $\omega^{\delta+n} \le \alpha < \omega^{\delta+n+1}$ then α is $\Delta^0_{\delta+2n}$ categorical, and is not Δ^0_β categorical for any $\beta < \delta + 2n$.

In the first part of talk we consider scattered linear orders such that ranks of them are constructive ordinals.

Theorem 1. *If a scattered linear order $\mathcal{L}$ has rank $\delta + n$, where δ is a constructive limit ordinal, n is finite ordinal, then $\mathcal{L}$ is relatively $\Delta^0_{\delta+2n}$ categorical.*

Theorem 2. *For any constructive ordinal $\delta + n \geq 2$, where δ is a constructive limit ordinal, n is finite ordinal and any β such that $3 \leq \beta \leq \delta + 2n$, and β is not a successor of a limit ordinal there exist a computable scattered linear order $\mathcal{L}$ with rank $\delta + n$ which is relatively Δ^0_β categorical and is not Δ^0_γ categorical for any $\gamma < \beta$.*

In the second part of talk we give a generalization of C.McCoy result about Δ^0_2-categorical linear order.

This work was supported by RFBR grant No. 18-31-00174.

References

[1] Goncharov S. S., Dzgoev V. D. Autostability of models // Algebra and Logic. – 1980. – V. 19. – Is. 1. – P. 45–58.

[2] Remmel J.B. Recursive categorical linear orderings // Proc. Am. Math. Soc. – 1981. – V. 83. – Is. 2. – P. 387–391.

[3] McCoy C. F. D. Δ^0_2-categoricity in Boolean algebras and linear orderings // Annals of Pure and Applied Logic. – 2003. – V. 119. – Is. 1–3. – Ðą. 85–120.

[4] McCoy C. O Δ^0_3-categoricity in linear orderings and in Boolean algebras // Algebra and Logic. – 2002. – V. 41. – Is. 5. – P. 531–552.

[5] Ash C. J. Recursive labelling systems and stability of recursive structures in hyperarithmetical degrees // Transactions of the American Mathematical Society. – 1986. – V. 298. – Is. 2. – P. 497–514.

[6] Bazhenov N. A. On degrees of autostability for linear orders and linearly ordered abelian groups // Algebra and Logic. – 2016. – V. 55. – Is. 2. – P. 133–155.

N.I. Lobachevsky Institute of Mathematics and Mechanics, Kazan Federal University, Kremlevskaya 18, Kazan, Russia
maxim.zubkov@kpfu.ru

Index by Author