LOG | COLLOQUIUM

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Model Checking: the Interval Way

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MODEL CHECKING

- Model checking: the desired properties of a system are checked against a model of the system
  - the model is a (finite) state-transition graph
  - system properties are specified by a temporal logic (e.g., LTL, CTL, CTL*, …)

- Distinctive features of model checking:
  - exhaustive verification of all the possible behaviours
  - fully automatic process
  - a counterexample is produced for a violated property

POINT-BASED VS. INTERVAL-BASED MC

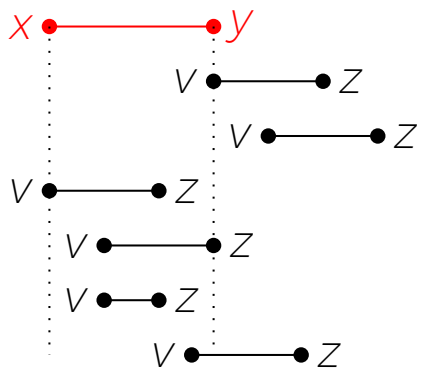
- Model checking (MC) is usually point-based:
  - properties express requirements over points (snapshots) of a computation (states of the state-transition system)
  - they are specified by means of point-based temporal logics such as LTL, CTL, and CTL*.

- Interval-based MC:
  - Interval-based properties express conditions on computation stretches
  - they are specified by means of interval temporal logics, which feature intervals as their basic ontological entities (e.g., HS)
    - ability to express: actions with duration, accomplishments, aggregations
    - applied to computational linguistics, artificial intelligence, temporal databases, formal verification

THE LOGIC HS

HS features a modality for each of the 13 Allen's ordering relations between pairs of intervals (except for equality)

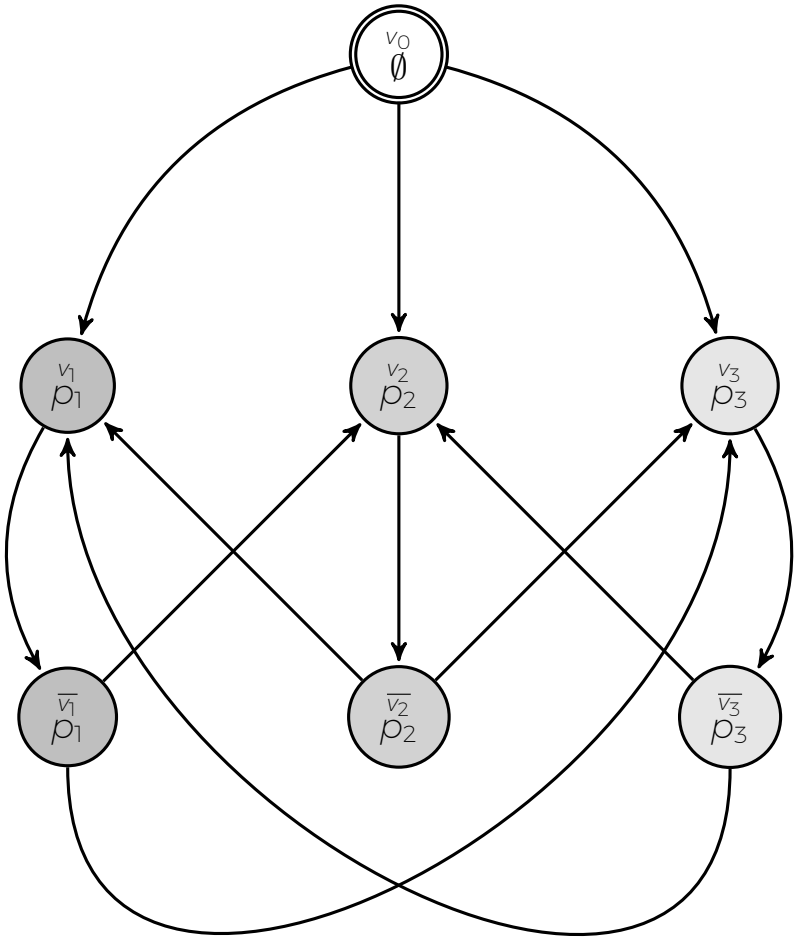
Allen rel.	HS	Definition	Example
meets	$\langle \mathsf{A} \rangle$	$[x,y]\mathcal{R}_A[v,z] \iff y = v$	
before	$\langle \mathsf{L} \rangle$	$[x,y]\mathcal{R}_L[v,z] \iff y < v$	
started-by	$\langle \mathsf{B} \rangle$	$[x,y]\mathcal{R}_B[v,z] \iff x = v \wedge z < y$	
finished-by	$\langle \mathsf{E} \rangle$	$[x,y]\mathcal{R}_E[v,z] \iff y = z \wedge x < v$	
contains	$\langle \mathsf{D} \rangle$	$[x,y]\mathcal{R}_D[v,z] \iff x < v \wedge z < y$	
overlaps	$\langle \mathsf{O} \rangle$	$[x,y]\mathcal{R}_O[v,z] \iff x < v < y < z$	



$$\psi ::= p \mid \neg\psi \mid \psi \vee \psi \mid \langle X \rangle \psi \mid \langle \overline{X} \rangle \psi \qquad X \in \{A, L, B, E, D, O\}.$$

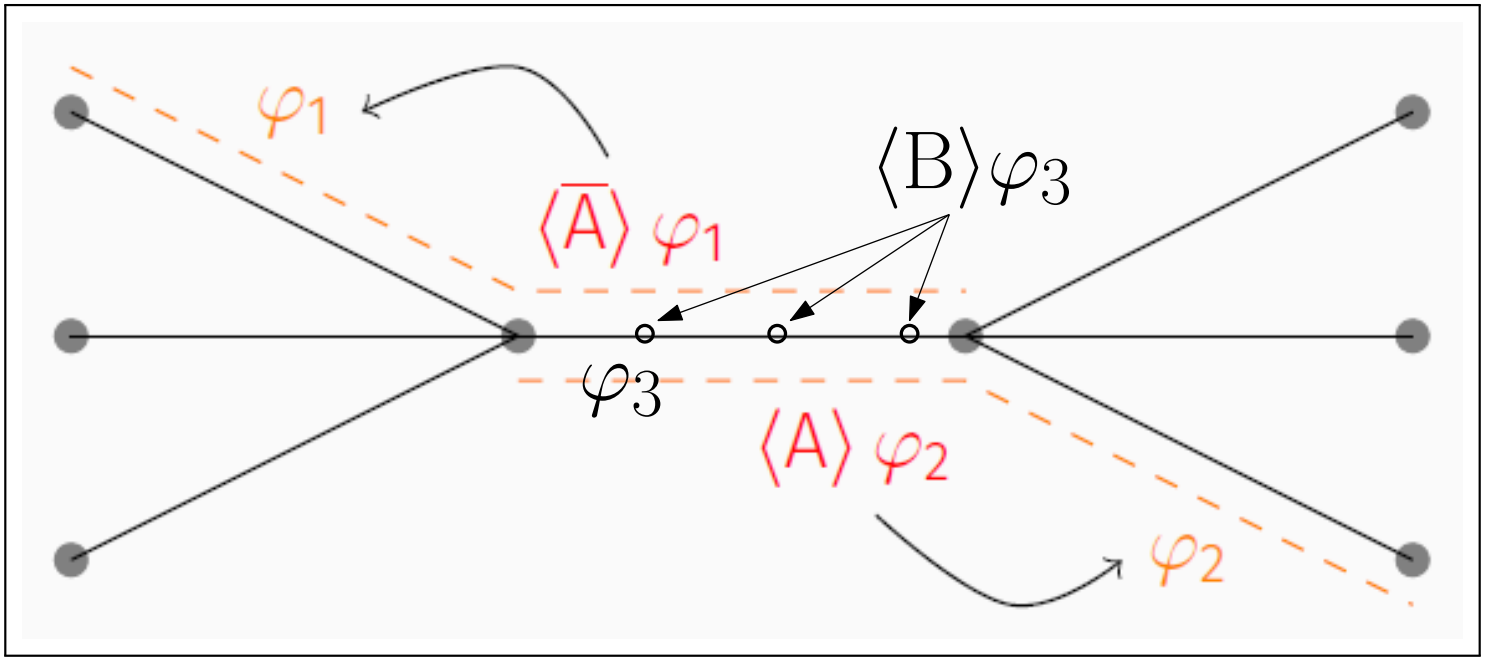
All modalities can be expressed by means of $\langle \mathsf{A} \rangle$, $\langle \mathsf{B} \rangle$, $\langle \mathsf{E} \rangle$ and their transposed modalities $\langle \overline{\mathsf{A}} \rangle$, $\langle \overline{\mathsf{B}} \rangle$, $\langle \overline{\mathsf{E}} \rangle$ only

KRIPKE STRUCTURES



- HS formulas are interpreted over (finite) state-transition systems whose states are labeled with sets of proposition letters (Kripke structures)
- An interval is a trace (finite path) in a Kripke structure

HS (STATE-BASED) SEMANTICS



- Branching semantics of past/future operators

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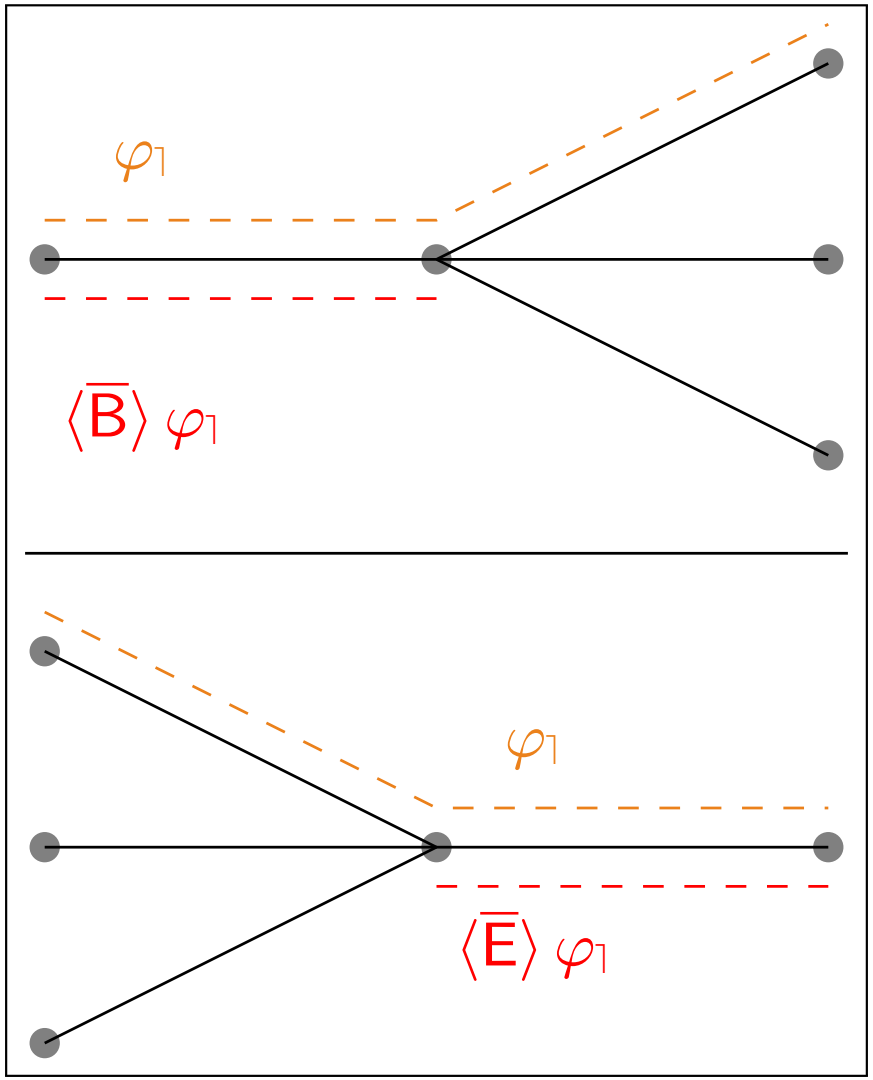

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HS (STATE-BASED) SEMANTICS AND MC

Truth of a formula ψ over a trace ρ of a Kripke structure $\mathcal{K} = (\mathcal{AP}, W, \delta, \mu, w_0)$:

- $\mathcal{K}, \rho \models p$ iff $p \in \bigcap_{w \in \mathsf{states}(\rho)} \mu(w)$, for any letter $p \in \mathcal{AP}$ (homogeneity assumption);

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  (labeling based on regular expressions, subsuming the others);

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- negation, disjunction, and conjunction are standard;
- $\mathcal{K}, \rho \models \langle \mathsf{A} \rangle \psi \ldots$;
- $\mathcal{K}, \rho \models \langle \mathsf{B} \rangle \psi \ldots$;
- $\mathcal{K}, \rho \models \langle \mathsf{E} \rangle \psi \ldots$;
- inverse operators $\langle \overline{\mathsf{A}} \rangle$, $\langle \overline{\mathsf{B}} \rangle$, $\langle \overline{\mathsf{E}} \rangle$

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MC

$\mathcal{K} \models \psi \iff$ for all *initial* traces ρ of $\mathcal{K}$, it holds that $\mathcal{K}, \rho \models \psi$

Possibly infinitely many traces!

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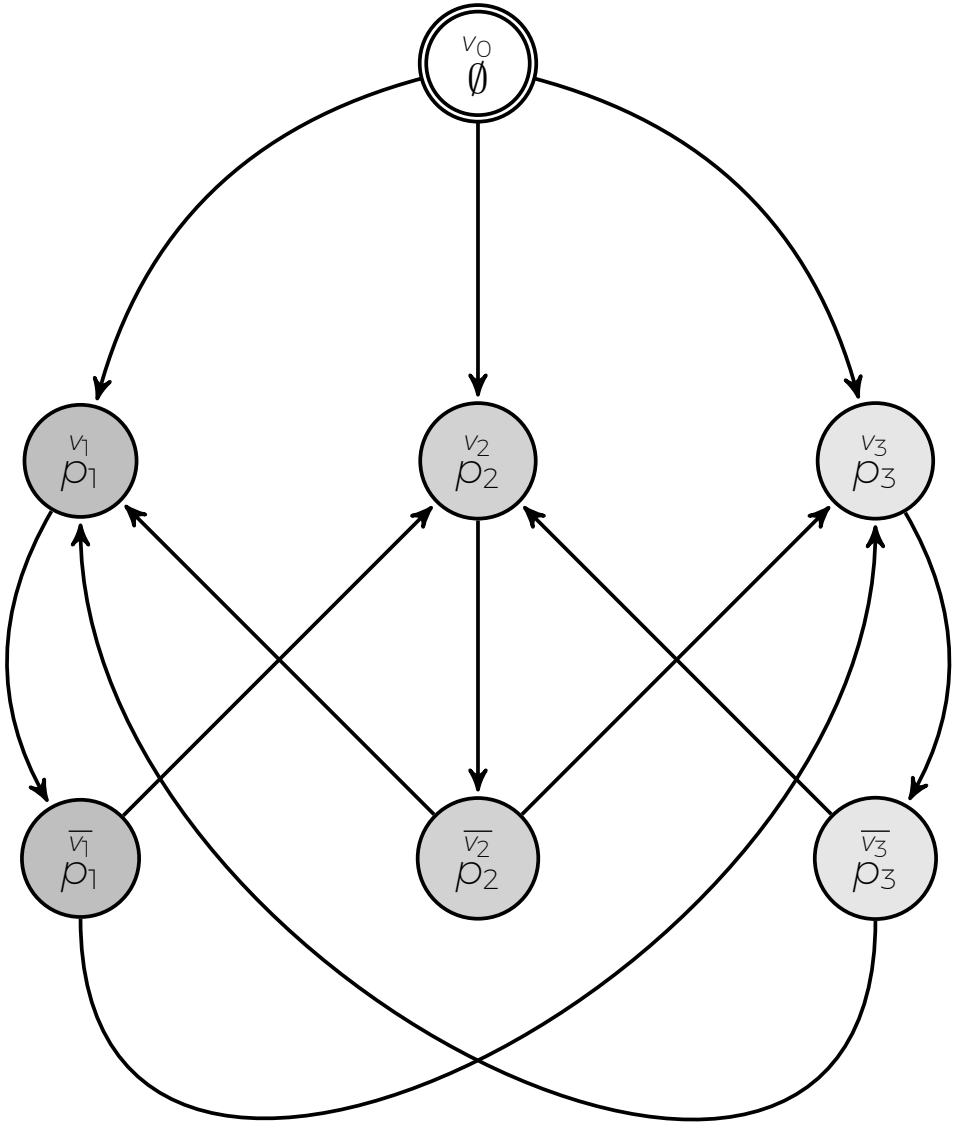
THE KRIPKE STRUCTURE $\mathcal{K}_{SCHED}$ FOR A SIMPLE SCHEDULER



A SHORT ACCOUNT OF $\mathcal{K}_{SCHED}$

$\mathcal{K}_{Sched}$ models the behaviour of a scheduler serving 3 processes which are continuously requesting the use of a common resource (easily generalizable to an arbitrary number of processes)

Initial state: v_0 (no process is served in that state)

In v_i and $\overline{v}_i$ the i-th process is served (p_i holds in those states)

The scheduler cannot serve the same process twice in two successive rounds:

- process i is served in state v_i, then, after "some time", a transition u_i from v_i to $\overline{v}_i$ is taken; subsequently, process i cannot be served again immediately, as v_i is not directly reachable from $\overline{v}_i$
- a transition r_j, with $j \neq i$, from $\overline{v}_i$ to v_j is then taken and process j is served

SOME PROPERTIES TO BE CHECKED OVER $\mathcal{K}_{SCHED}$

Validity of properties over all reachable computation intervals can be forced by modality $[E]$ (they are suffixes of at least one initial trace).

- In any computation interval of length at least 4, at least 2 processes are witnessed (YES: no process can be executed twice in a row)

$$\mathcal{K}_{Sched} \models [E](\langle \mathsf{E} \rangle^3 \top \rightarrow (\chi(p_1, p_2) \vee \chi(p_1, p_3) \vee \chi(p_2, p_3))),$$

  where $\chi(p, q) = \langle \mathsf{E} \rangle \langle \overline{\mathsf{A}} \rangle p \wedge \langle \mathsf{E} \rangle \langle \overline{\mathsf{A}} \rangle q$.

- In any computation interval of length at least 11, process 3 is executed at least once (NO: the scheduler can postpone the execution of a process ad libitum—starvation)

$$\mathcal{K}_{Sched} \not\models [E](\langle \mathsf{E} \rangle^{10} \top \rightarrow \langle \mathsf{E} \rangle \langle \overline{\mathsf{A}} \rangle p_3).$$

- In any computation interval of length at least 6, all processes are witnessed (NO: the scheduler should be forced to execute them in a strictly periodic manner, which is not the case)

$$\mathcal{K}_{Sched} \not\models [E](\langle \mathsf{E} \rangle^5 \rightarrow (\langle \mathsf{E} \rangle \langle \overline{\mathsf{A}} \rangle p_1 \wedge \langle \mathsf{E} \rangle \langle \overline{\mathsf{A}} \rangle p_2 \wedge \langle \mathsf{E} \rangle \langle \overline{\mathsf{A}} \rangle p_3)).$$

MC: THE KEY NOTION OF BE_K-DESCRIPTOR

- The BE-nesting depth of an HS formula ψ ($\mathsf{Nest}_{\mathsf{BE}}(\psi)$) is the maximum degree of nesting of modalities B and E in ψ
- Two traces ρ and ρ' of a Kripke structure $\mathcal{K}$ are k-equivalent iff:

  $\mathcal{K}, \rho \models \psi$ iff $\mathcal{K}, \rho' \models \psi$ for all HS formulas ψ with $\mathsf{Nest}_{\mathsf{BE}}(\psi) \leq k$

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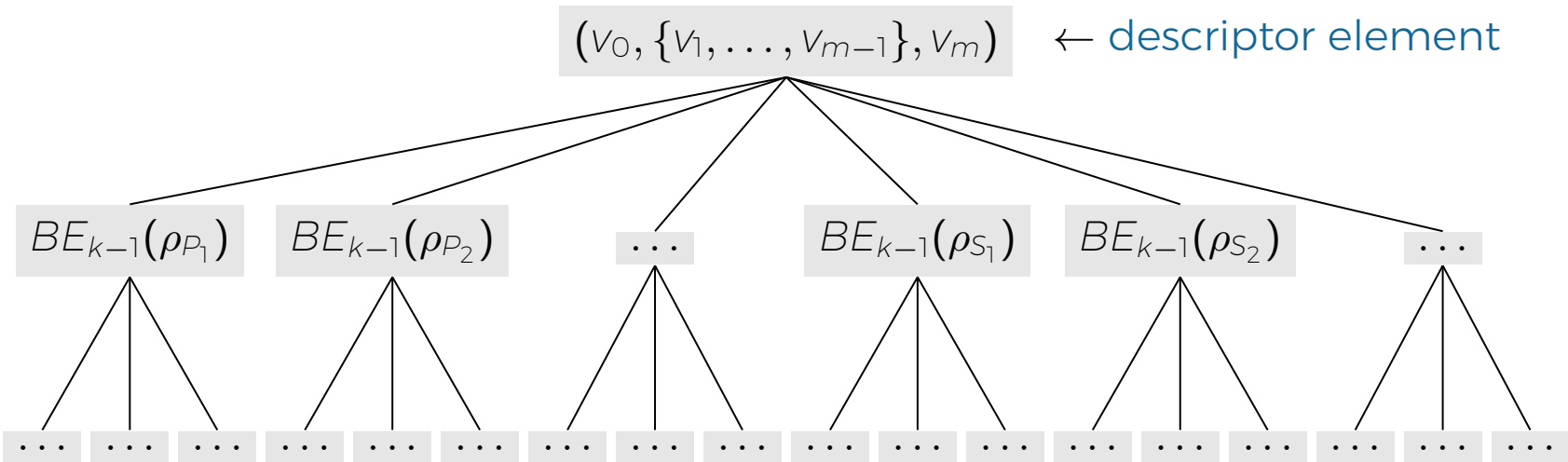
For any given k, we provide a suitable tree representation for a trace, called a BE_k-descriptor

MC: THE KEY NOTION OF BE_K-DESCRIPTOR

The BE_k-descriptor for a trace $\rho = v_0 v_1 \ldots v_{m-1}\ v_m$, denoted $BE_k(\rho)$, has the following structure:



↑ $\rho_{P_1}, \rho_{P_2}, \ldots$ prefixes of ρ

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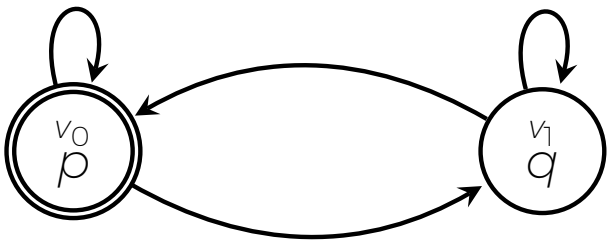
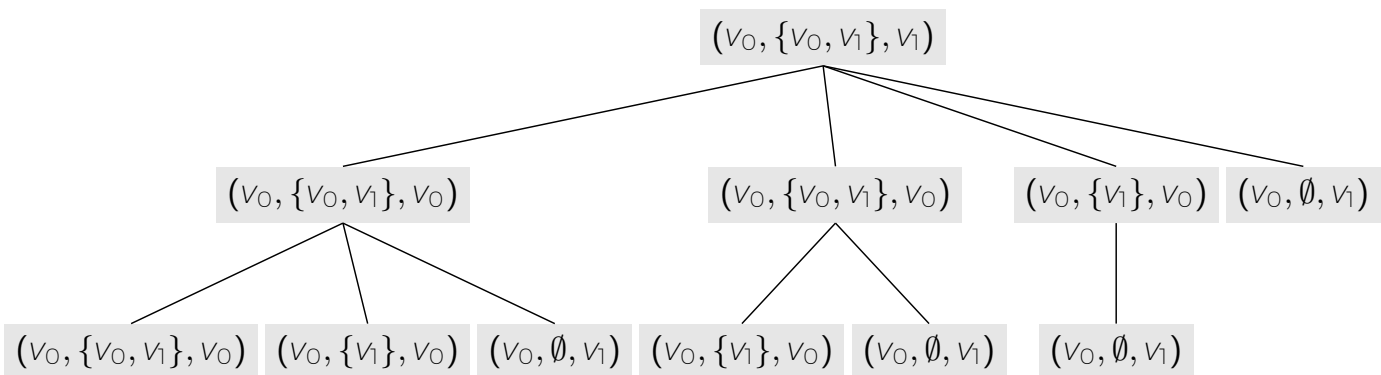
Remark: the descriptor does not feature sibling isomorphic subtrees

AN EXAMPLE OF A BE_2-DESCRIPTOR



The BE_2-descriptor for the trace $\rho = v_0 v_1 v_0^4 v_1$ (for the sake of readability, only the subtrees for prefixes are displayed and point intervals are excluded)



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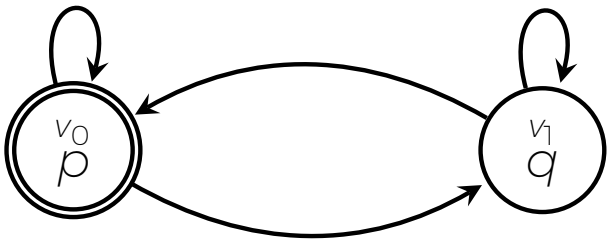

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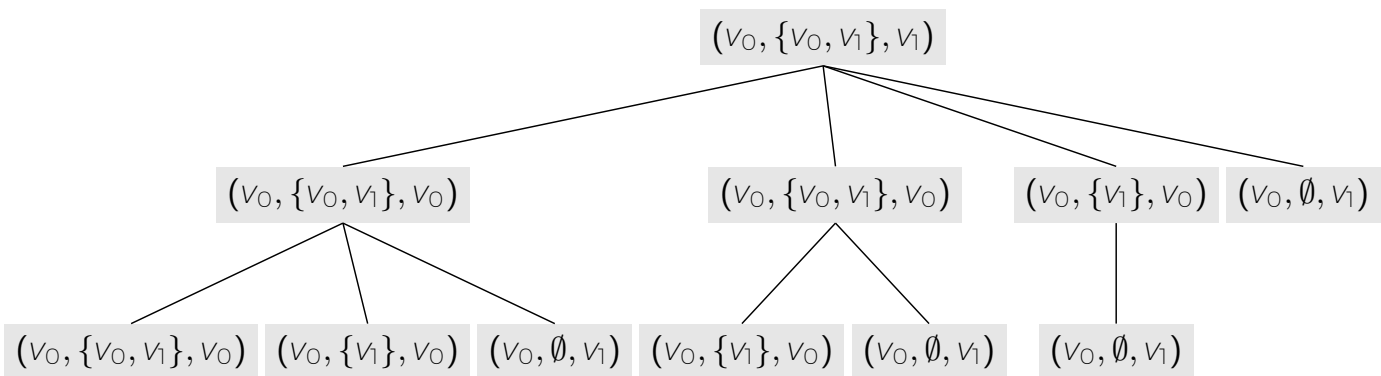

Remark: the subtree to the left is associated with both prefixes $v_0 v_1 v_0^3$ and $v_0 v_1 v_0^4$ (no sibling isomorphic subtrees in the descriptor)

DECIDABILITY OF MC FOR FULL HS

FACT 1: For any Kripke structure $\mathcal{K}$ and any BE-nesting depth $k \geq 0$, the number of different BE_k-descriptors is finite (and thus at least one descriptor has to be associated with infinitely many traces)

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FACT 2: Two traces ρ and ρ' of a Kripke structure $\mathcal{K}$ described by the same BE_k-descriptor are k-equivalent

DECIDABILITY OF HS MC

Theorem

The MC problem for full HS *over Kripke structures is decidable (nonelementary algorithm)*

Reference

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Theorem

The MC problem for BE *over Kripke structures is* **EXPSPACE***-hard*.

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THE LOGIC $A\overline{A}B\overline{B}\overline{E}$

Let us consider the case of the logic $A\overline{A}B\overline{B}\overline{E}$, which is obtained from full HS ($A\overline{A}BE\overline{B}\overline{E}$) by removing modality $\langle \mathbf{E} \rangle$

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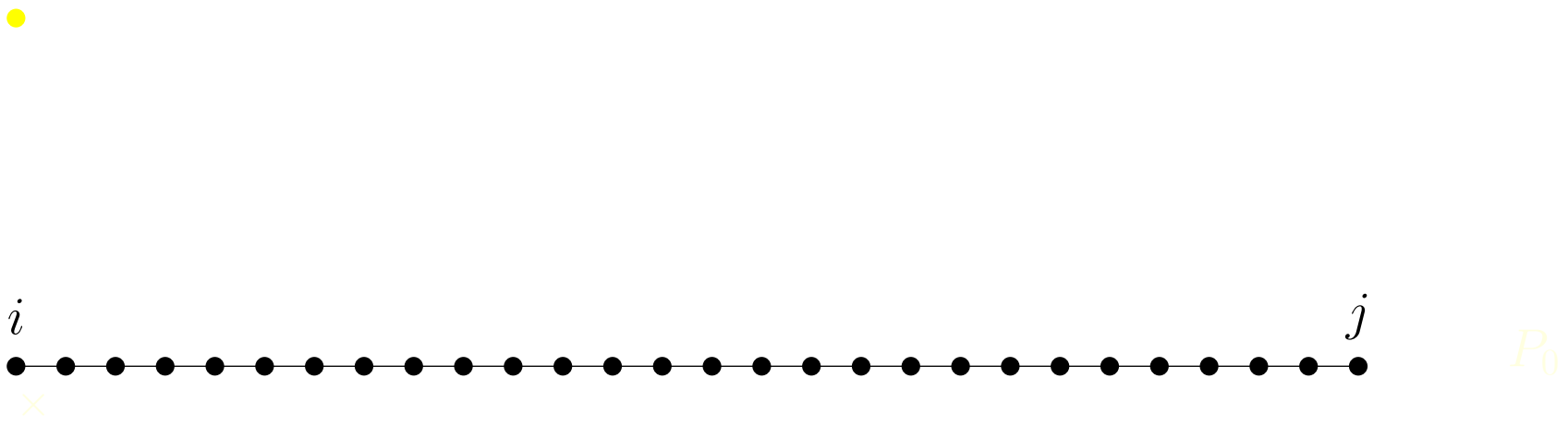
A high-level account of the solution:

- we can restrict our attention to prefixes (B_k-descriptors suffice)
- the size of the tree representation of B_k-descriptors is larger than necessary (redundancy) and it prevents their efficient use in MC algorithms
- a trace representative can be chosen to represent a (possibly infinite) set of traces associated with the same B_k-descriptor
- a bound, which depends on both the number $|W|$ of states of the Kripke structure and the B-nesting depth h of the formula to check, can be given to the length of trace representatives

H-PREFIX SAMPLING



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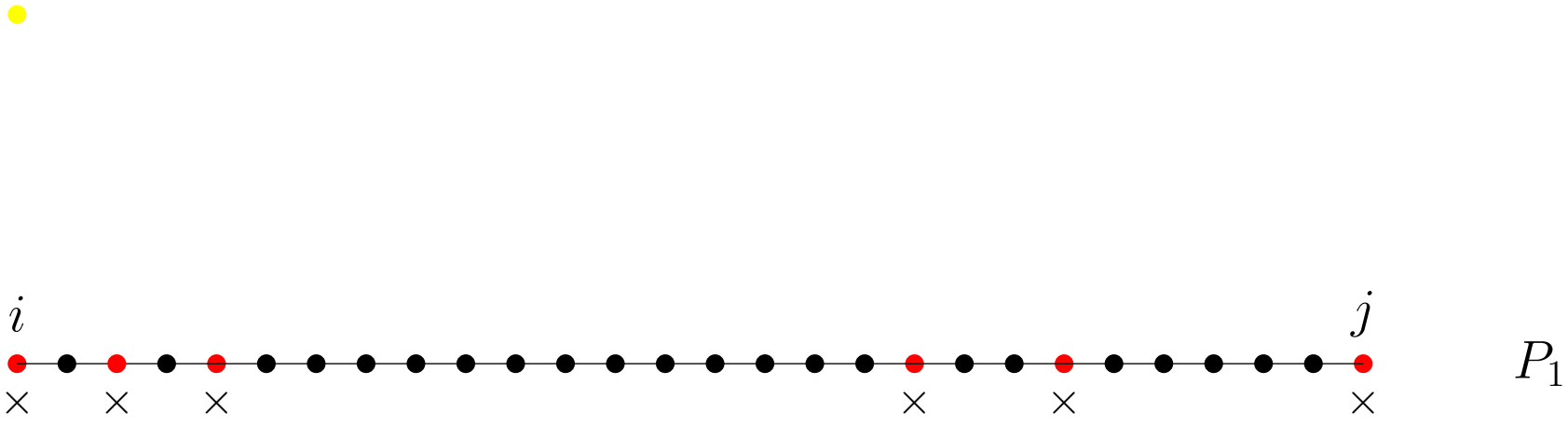

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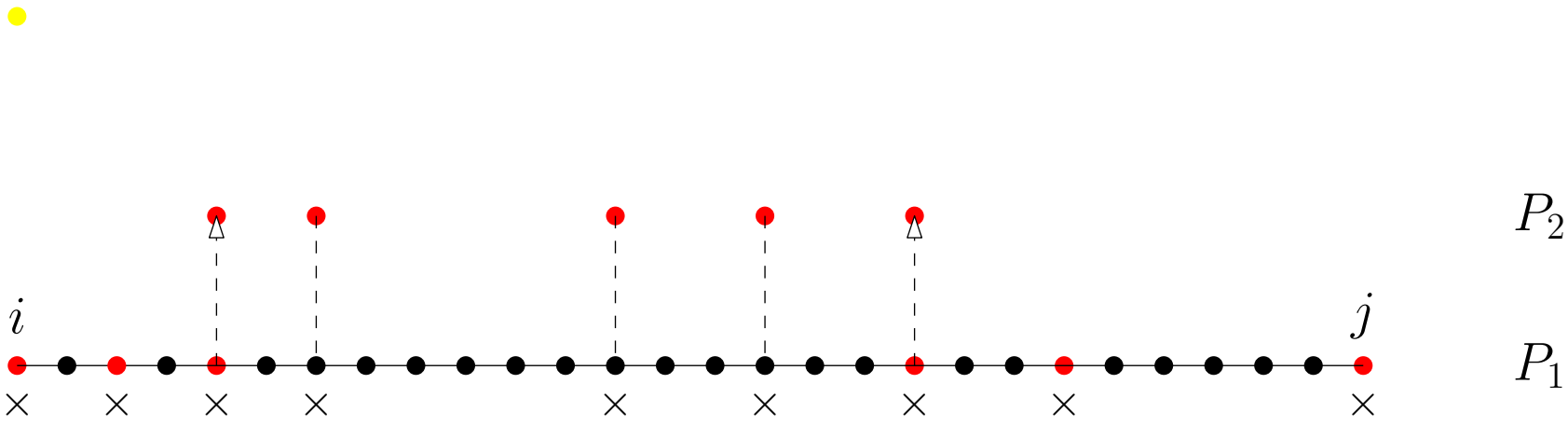

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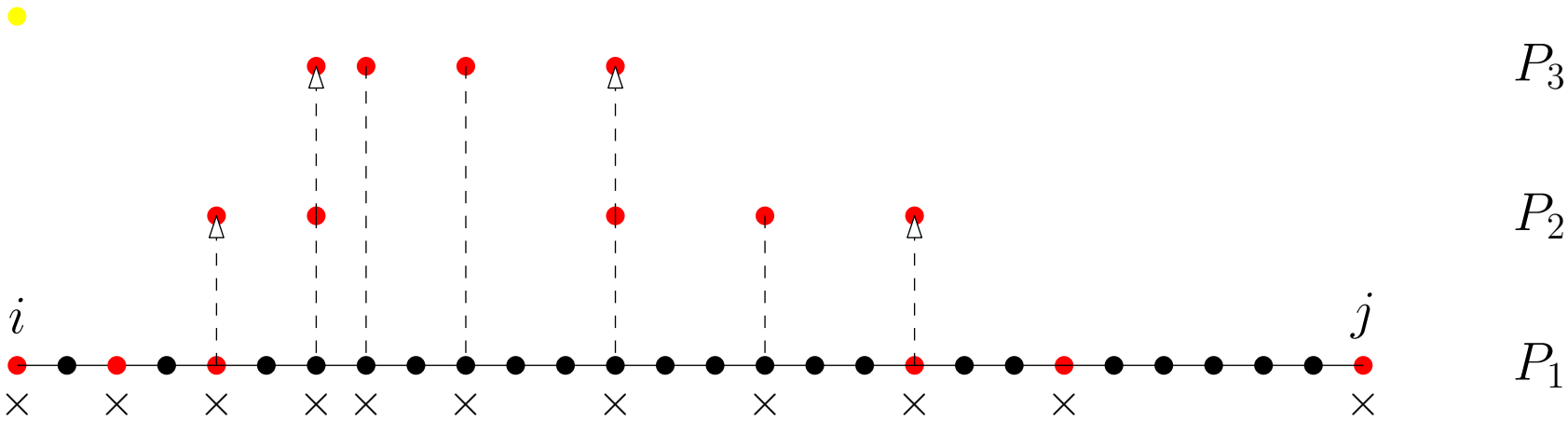


A SMALL-MODEL (TRACE) RESULT

- From a trace ρ, we can derive an h-equivalent trace ρ' in this way:
  1. we first compute the $(h+1)$-prefix sampling P_{h+1} of ρ;
  2. then for all the pairs of consecutive ρ-positions $i, j \in P_{h+1}$, we replace each $\rho(i,j)$ by another trace with no repeated occurrences of any state, except at most the first/last ones (hence no longer than $(|\mathcal{K}|+2)$).
- ρ and ρ' can be proved to be h-equivalent
- By the previous bound on $|P_{h+1}|$, we have $|\rho'| \leq (|\mathcal{K}|+2)^{h+2}$.

AN EXPSPACE MC ALGORITHM FOR $A\overline{A}B\overline{B}\overline{E}$

Theorem (Small model/trace property)

Given a trace ρ, we can derive its trace representative ρ', $\mathsf{Nest}_B(\psi)$-equivalent to it, such that $|\rho'| \leq (|\mathcal{K}| + 2)^{\mathsf{Nest}_B(\psi)+2}$

Algorithm 1 $\texttt{ModCheck}(\mathcal{K}, \psi)$

1: $h \leftarrow \mathsf{Nest}_B(\psi)$
2: for all initial traces ρ' with $|\rho'| \leq (|\mathcal{K}| + 2)^{h+2}$ do
3: if $\texttt{Check}(\mathcal{K}, h, \psi, \rho') = 0$ then return 0: "$\mathcal{K}, \rho' \not\models \psi$" ◁ *Counterex* ✗
return 1: "$\mathcal{K} \models \psi$" ◁ *MC OK* ✓

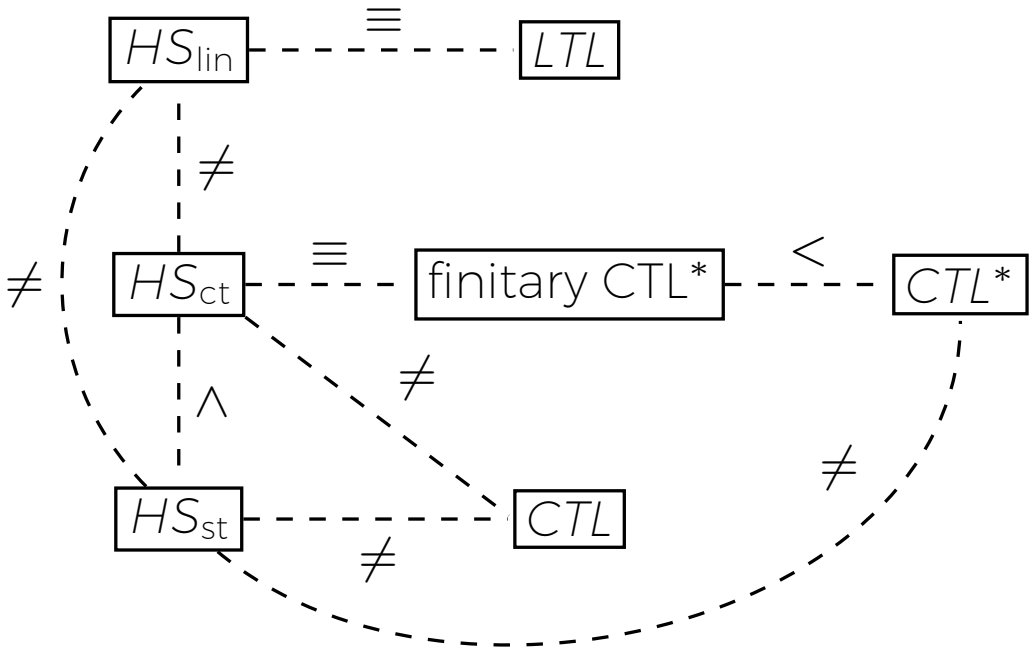
Reference

L. Bozzelli, A. Molinari, A. Montanari, A. Peron, and P. Sala. Interval temporal logic model checking based on track bisimilarity and prefix sampling.
In *ICTCS*, pages 49–61, 2016

COMPLEXITY RESULTS

	Homogeneity
Full *HS*, BE	non-elementary **EXPSPACE**-hard
$A\overline{A}B\overline{BE}$, $A\overline{A}E\overline{BE}$	$\in$ $\mathbf{AEXP}_{\mathbf{Pol}}$ **PSPACE**-hard
$A\overline{ABE}$	**PSPACE**-complete
$A\overline{A}B\overline{B}$, $B\overline{B}$, $\overline{B}$, $A\overline{A}E\overline{E}$, $E\overline{E}$, $\overline{E}$	**PSPACE**-complete
$A\overline{A}B$, $A\overline{A}E$, AB, $\overline{A}E$	$\mathbf{P}^{\mathbf{NP}}$-complete
$A\overline{A}$, $\overline{A}B$, AE, A, $\overline{A}$	$\in$ $\mathbf{P}^{\mathbf{NP}[O(\log^2 n)]}$ $\mathbf{P}^{\mathbf{NP}[O(\log n)]}$-hard
Prop, B, E	**co-NP**-complete

EXPRESSIVENESS RESULTS (UNDER HOMOGENEITY)



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