Natural language variants of universal quantification in first order modal logic

D. Catta A. Mari M. Parigot C. Retoré

LIRMM-Université de Montpellier, IJN-CNRS, IRIF-CNRS

The problem

- French has three wordings of universal quantification: *chaque* (singular; $\sim$ *each*), *tout* (singular; $\not\sim$ *every*) and a third one, *tous les* (plural; $\sim$ *all*)
- We just discuss the two first ones because they illustrate a general phenomenon: universals quantifier may tolerates exceptions.
- we will first expose the linguistic characteristic of the two quantifiers and then propose a formalisation in first order modal logic.
- this formalisation is indeed more general: it permits to capture the phenomena of *prima facie* principles

Tout and chaque: a first approximation

- **Tout** is a universal quantifier that tolerates exceptions
- **Chaque** is a universal quantifier that does not tolerates exceptions

Tout

- **Tout** is a universal quantifier that tolerates exception

1 Tout oiseau vole. ($\sim$ Any bird flies)

Exceptions to the statement can be raised without invalidating the statement itself

1.1 Sauf les pingouins ($\sim$ Except penguins)

1.2 Sauf ce pigeon blessé. ($\sim$ Except this wounded pigeons)

Chaque

- **Chaque** is a universal quantifier which *does not* tolerate exceptions. It conveys the information that the speaker can concretely verify the truth of the assertion.

  2 Chaque oiseau vole ($\sim$ Each bird flies)

  The following is a valid refutations

  2.1 Sauf ce pigeon blessé. ($\sim$ Except this wounded pigeons)

Another linguistic example

Tout does not tolerate accidental properties, unless a rule is contextually triggered.

3 Chaque enfant est habillé en rouge ($\sim$ Each child is wearing red)

4 ?? Tout enfant est habillé en rouge ($\sim$ Any child is wearing red)

5 Tout enfant de l'école élementaire Pascal est habillé en rouge. ($\sim$ every child of the Pascal elementary school is dressed in red)

Tout and implicit quantification

- The *meaning* of *tout* appear to be similar to the *implicit* quantification common in proposition taken from the domain of law and ethics.

Fifth Commandment: Thou shalt not kill

Exceptions

1. Justified killing: due consequence for crime
2. Justified killing: in warfare
3. Justified killing: intruder in the home

Exceptions do not invalidate the general rule.

Tout and chaque: linguistic categorisation

- **Descriptive quantifiers** The enunciation of a sentence involving a descriptive quantifier convey information about the world. **Chaque** belongs to this class of quantifiers.

- **Definitional/prescriptive quantifiers**: the enunciation of a sentence involving a quantifier of this type convey information about how the world should be like according to an assessor (or a group or the whole community). **Tout** belongs to this class of quantifiers.

A formal treatment of *tout* and *chaque*

- The usual approach to this phenomenon - Krifka (1995) for a survey- is to formally model *definitional quantifiers* in first order modal logic by imposing normalcy conditions on worlds i.e., a metric that measure the similarity between a fixed world w in the model and the other worlds w_i. The idea is that a sentence having a *definitional quantifier* $\mathbf{Q}$ as main connective e.g., $\mathbf{Q}x.A(x)$ is true whenever exceptions, i.e. $\neg A(x)$, are true only in worlds that are "far away" from w according to the similarity measure.
- We are not sympathetic with this approach. We are not interested in modelling the fact that a particular rule admits exceptions. We would like to model a general and common fact: a universal quantifier that admits exceptions.

The big picture

We will work in Kripke-frames. In our setting

- *Chaque* is a quantification in a particular world
- *Tout* is a quantification over all the worlds.

If one would like to model a particular situation, e.g. in law or linguistic, she will choose a particular logic, a particular structure, etc...

Disclaimer

In the following we focus on constant domain model because they are simpler to get. However our approach works as well with varying domain models.

FOL modal logic 1

Definition (formula)

$$A, B := \; R^n(x_1, \ldots x_n) \; |\neg A \mid A \wedge B \mid \Box A \mid \forall x A$$

Definition (Augmented Frame)

A structure $< G, R, D >$ is a constant domain augmented frame if $< G, R >$ is a frame and D is a non empty set called *the domain of the frame*.

Definition (Interpretation)

I is an interpretation in a constant domain augmented frame $< G, R, D >$ if I assigns to each n-place relation symbol R and to each possible world $w \in G$ some n-place relation on the domain D of the frame

Definition (Model)

A constant domain first-order model is a structure $M =< G, R, D, I >$ where $< G, R, D >$ is a constant domain augmented frame and I is an interpretation in it.

FOL modal logic 2

Definition (Valuation)

Let $M = < G, R, D, I >$ be a constant domain first-order model. A valuation in the model M is a mapping v that assigns to each free variable x some member $v(x)$ of the domain D of the model.

Definition (Variant)

Let v and v' be two valuations. We say v' is an x-variant of v if v and v' agree on all variables except possibly the variable x.

Definition (Truth in a Model)

Let $M = < G, R, D, I >$ be a constant domain first-order modal model. For each $w \in G$, and each valuation v in M:

1. if R is a n-place relation $M, w \models_v R(x_1, \dots x_n)$ provided $(v(x_1), \dots v(x_n)) \in I(R, w)$
2. $M, w \models_v \neg A \iff M, w \not\models_v A$
3. $M, w \models_v A \wedge B \iff M, w \models_v A$ and $M, w \models_v B$
4. $M, w \models_v A \iff$ for every $k \in G$ if wRk then $M, k \models_v A$
5. $M, w \models_v \forall x A \iff$ for every x-variant v' of v in M $M, w \models_{v'} A$

Tout and Chaque: a first distinction

Chaque is a descriptive quantifier that convey information about the word that the speaker inhabits and it does not admits exceptions. Thus we propose **Chaque** $x.A(x) := \forall x.A(x)$ i.e. chaque speaks about *truth in a particular world*

Tout Is a definitional quantifier. It convey information about how the "world" should be according to the speaker. It is tempting to model this simply as $\Box(\forall x.A(x))$. However this is far from capture the **admits exceptions** clause

Refining tout: a dialogical idea 1

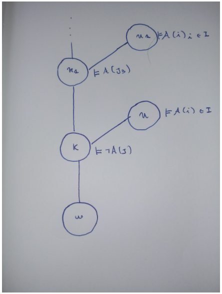
We must model the fact that tout admits exceptions. We explain our formalisation trough a dialogical metaphor.

- Imagine that someone - say Alice- affirms a sentence having a definitional quantifier as main connective e.g., $\mathbf{Q}xA$
- Imagine that someone - say Bertrand- proposes some exceptions i.e. some individuals t_i such that $\neg A(t_i)$
- Imagine also that for every Bertrand intervention Alice is able to reply to the intervention explaining why that exception does not count

The proposed solution

$$\textbf{Tout}\ x.A := \Box\Diamond(\forall x A)$$

- This is a *persistent* universal quantifier i.e., the truth of $\forall x A$ passes unharmed through exceptions
- The number of exceptions is possibly infinite as the following shows:



- One inherit the proof theory for modal logic.

Further works

1. We are currently trying to find direct rules for the quantifier **tout** $x.A(x)$. (simpler than the one inherited from modal logic)

2. How can we define models for specific domain of knowledge where one uses this quantifier e.g. law?

Self-explaining

Thank you for you attention