Reading Bi-logic in first order language

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Logic Colloquium 2018

Finite vs Infinite representations in Freud's essay On Aphasia

One of the pillars of the freudian psychoanalysis, the relationship between thought and language, is underlined by Freud's distinction between word-presentations - the mental images of words, and thing-presentations - the representations of actual objects. This little-known theorization was first postulated by Freud in one of his earliest works - On Aphasia (Freud, 1891).

On the basis of the distinction already proposed by Stuart Mill in his Logik, he proposed to consider the

- **word-presentation** (Wortvorstellung) the **finite** linguistic form, represented by a **closed term**;

- **thing-presentation** (Objektvorstellung or Sachvorstellung) represented by the corresponding **open, infinite term**.

Therefore, on the topic of the relationship between language and thinking, Freud promoted the idea that a thought precedes language: thought is initially unconscious and concerned with the sense impressions left by objects, when it later becomes conscious, it does so only by linking with the mental representations of the words.

He felt the need to consider both **closed** and **open** representations in order to understands this. In the development of psychoanalytic theory he kept the assumption that open, infinite thing representations are always to be connected to word-presentations in order to allow access to conscious processing.

Open, infinite, thing-presentations would be therefore, in themselves, always unconscious.

Word-presentations involve the linking of a conscious idea to a verbal stimulus, are associated with the secondary process, and are oriented towards reality.

Thing-presentations are essentially pre- or non- verbal images of objects, they are associated with the primary process, and are not necessarily connected with reality.

Matte Blanco's Bi-Logic Model

The Chilean psychoanalyst I. Matte Blanco (1907-1995) wanted to further develop the Freudian psychoanalytic theory, and he developed his own view of the human mind with the help of the notions from the field of Logic (he largely refers to the concepts of Set Theory, in particular to the Cantorian notion of Infinite Set).

He proposed a system, the so called Bi-Logic Model, which describes the human thinking as underlined by a mixture of two modes - the conscious and unconscious ones. And according to him, these modes of thinking/being can be explained with the help of different logical systems. (Matte Blanco, 1975).

He describes mental functions in terms of the entanglement of two different ways of functioning, corresponding to the Freudian Primary and Secondary Processes.

According to this theoretical proposal two opposite and apparently irreducible and contradictory ways of being do coexist in mental life:

the asymmetric and heterogenic mode, following the rules of classical reasoning

the symmetric and homogenic mode, which can be described as a logical system operating on the basis of two fundamental principles.

The generalization principle

The sistem Ucs treats an individual thing (person, object, concept) as if it were a member or element of a set or class which contains other members; it treats this class as a subclass of a more general class, and this more general class as a subclass or subset of a still more general class, and so on. (Matte Blanco, 1975, p.38)

The symmetry principle

The system Ucs treats the converse of any relation as identical with the relation. In other words, it treats asymmetrical relations as if they were symmetrical. (Matte Blanco, 1975, p.38).

On the antinomy of thought

In the book entitled *Thinking, Feeling and Being*, Matte Blanco explores the mutual relationships between the asymmetric and symmetric logical modes. He concludes that

the heterogenic mode is the realm of the logical. The symmetric mode is the realm of the illogical. The Freudian Unconscious is the realm of bi-logical structures and, as such, the realm of antinomies.

He then clarifies that the former conclusion was derived from the perspective of classical logic, as it is used in thinking and reasoning.

He then explains that *If, instead, the question could be seen in the light of a unitary super-logic, which is not yet available (...), the conclusion just mentioned might no longer be true.* (Matte Blanco, 1998, p.82).

Bi-logic and first order language

As a basis: we model Bi-logic (Symmetric Mode vs. Bivalent Mode) by distinguishing infinite from finite.

To this aim: we perform an analysis of first order language in terms of the representations it can produce. We adopt the method of introducing logical connectives via equations (as in basic logic).

We first see an interpretation of "infinite" and how infinite is related to symmetric and finite to asymmetric.

Then we see that the propositions of the symmetric mode can be conceived as universal propositions on paticular domains: infinite singletons.

Then we suggest the role of modalities in order to put the two modes together (how to import the infinite into the finite).

Infinite sets

The representation of a "finite" set can be given by enumeration:

$$D = \{t_1, \dots, t_n\}$$

"finite" means that I can distinguish its elements and then I can count n elements of D.

Let us denote the elements of D by closed terms t_i.
Let $z \in D$ be an "unknown" element: then $z = t_i$ for some i.

Formally it should be

$$z \in D \vdash z = t_1 \vee \cdots \vee z = t_n$$

We can consider a formal system which does not include such an assumption: that system cannot conclude that D is finite.

Infinite sets and symmetric mode

In the symmetric mode every relation is symmetric

The class of sets characterized by the symmetric mode is the class of singletons.

Singletons are usually finite: one element u, and $D = \{u\}$.

This means that we assume the equivalence $z \in \{u\} \equiv z = u$ that is true by extensionality.

This means that we "recognize" the element u.

But, in the unconscious, sets are infinite: we need infinite singletons!

We need a domain V equipped with a "unique" (non recognizable) element, that is an (infinite) singleton.

We need to say "singleton" intensionally, putting

$$(\forall x \in V)A(x) = (\exists x \in V)A(x)$$

for every A.

The quantifier $\forall$ is introduced considering the metalinguistic definition

$$\Gamma \textit{ yields } A(z) \textit{ forall } z \in V$$

where Γ is a set of hypothesis which do not depend on z.

Then, formally, we write it

$$\Gamma, z \in V \vdash A(z)$$

where z a variable of the language and then $z \in V$ as a formal assumption.

We consider the following equation, that can close the formula A with respect to the variable z and provides the definition of $\forall$:

$$\Gamma \vdash (\forall x \in V)A(x) \quad \equiv \quad \Gamma, z \in V \vdash A(z)$$

The quantifier $\exists$ has, formally, a *symmetric* definition:

$$(\exists x \in V)A(x) \vdash \Delta \quad \equiv \quad A(y) \vdash \Delta, (y \in V)^{\perp}$$

where $(y \in V)^{\perp}$ is a *dual* formula which represents negation.

When V is a singleton, namely assuming the equivalence $(\forall x \in V)A(x) \equiv (\exists x \in V)A(x)$ for every A, and putting the two definitions together one has the consequence

$$z \in V, A(y) \vdash A(z), (y \in V)^{\perp}$$

for every A.

Let us read it "symmetrically". Then $(y \in V)^{\perp}$ should have the status of a membership itself. Namely, one characterizes an (infinite) singleton $V^{\perp}$: "the complement of V".

Two cases are possible:

FINITE CASE: extensionality $V = \{u\}$.
Then $x \in V$ means $x = u$, and then $(x \in V)^{\perp}$ means $x \neq u$ that should be be considered in turn another equality: $x = u^{\perp}$.
We characterize a *dual* element and we have BIVALENCE.

INFINITE CASE: since no element of V is characterized, no dual element is characterized.

We can clearly see here Freud's dichotomy between **'word-presentation'** given by the finite linguistic closed form vs **'thing-presentation'**, the corresponding open term.

In the infinite case, that is when the representation has the form of a thing-presentation, perfect symmetry is obtained putting
$V^{\perp} = V$.
This means: no negation, no logical consequence!

We need to refer to the quantum model: We consider the spin observable (it is bivalent!)

We perform the measurement of the spin with respect to the z axis. We find two eigenvalues: "up" and "down". They are "opposite" (*dual*) since they are switched by an operator, that is considered the negation operator.

Then we consider the eigenvalues (fixed points) of the negation operator. By definition, they are not dual with respect to negation. No "objective value" can be attributed to them. We could find a value if we could measure with respect to the x axis, but this is incompatible. Then our fixed points are infinite singletons. Actually, we can think that they are "inhabited" by the random variable given by their measurement with respect to the z axis.

Thinking, feeling and being - How to link the two modes?

The need to go beyond the characterization of the two modes is clearly expressed by Matte Blanco, particularly in "Thinking, feeling and being".

The idea is already present in Freud, since, in his idea, affects are forced into the representational domain, whereas their original and basic expression is in *actions*.

We have seen that the transition from symmetric to bivalent is provided by an identification: $z \in V$ becomes $z = u$.

We try to obtain a kind of "abstract identification". Namely a way of giving the singleton a "value" even if we cannot declare it, and a way of "closing" a formula even without variables.

Let us consider again the definition of quantifier:

$$\Gamma \vdash (\forall x \in V)A(x) \ \equiv \ \Gamma, z \in V \vdash A(z)$$

where Γ is closed with respect to the free variable z.

Let us generalize this fact: we now consider a *closed* set of assumptions Γ, that means "closed w.r.t. any variable on any infinite singleton". We write $\Box$ for such a closure. Then we generalize the equation which defines the quantifier, putting:

$$\Box\Gamma \vdash \Box A \ \equiv \ \Box\Gamma \vdash A$$

where variables, closed terms and domains have been eliminated.

The solution are the rules of the modal system S_4.

Notice that the definition of $\Box$ could be equally read as a generalization of the following definition, that considers parameters at the metalanguage rather than variables in the object language and defines a "finite" quantifier $\forall_\omega$, characterized by an ω-rule:

$$\Gamma \vdash (\forall_\omega x \in V)A(x) \equiv \Gamma \vdash A(t_i) \textit{ forall } t_i \in V$$

Then the modality $\Box$ describes propositions in the middle between the infinite objects given by $\forall$ and the finite objects given by $\forall_\omega$.

Kurt Gödel defined the modality in order to introduce an "infinite" provability predicate (w.r.t. the provability by finite methods) to the aim of avoiding incompleteness. Then, he showed that one can define intuitionistic logic adding $\Box$ to classical propositional logic, and that intuitionistic logic is infinite-valued.

Then, by means the modal operator, one can add an infinite content to bivalent logic.

On the other side, considering symmetric propositions, and the operator $\Box$, one can define a negation putting

$$\neg A \equiv \Box A \supset \bot$$

(an interpretation of the constants $\bot$ and $\supset$ is possible in the quantum model).

The modal operator can "create" logic from the symmetric mode, keeping its infinite content by means of $\Box$.

The modal operator $\Box$ is interpreted as "necessary". We could attribute a normative value to necessity. Then normativity would get an intermediate status between the infinite and finite mode.

There is consistency between the logical features of the operator $\Box$ so introduced, and the character of the super-ego, described by Freud in "The ego and the id". According to Freud, the super-ego, formed before the characterization of the parental figures, has the form of an abstract authority.

Concerning its role between the ego and the id, Freud stresses that

...Thus the super-ego is always close to the id and can act as its representative vis-a-vis the ego. It reaches deep down into the id and for that reason is farther from consciousness than the ego is.

...the super-ego knew more than the ego about the unconscious id.

S. Freud, *Zur Auffassung der Aphasien.Eine kritische Studie* (1891)

S. Freud, *Das Ich und das Es* (1923)

I. Matte Blanco, *The Unconscious as Infinite Sets* (1975)

I. Matte Blanco, *Thinking, Feeling and Being* (1988)

Thank you