Forcing in differential topology of 4-manifolds and physical cosmology

Jerzy Król*
with Torsten Asselmeyer-Maluga**
Krzysztof Bielas* and Paweł Klimasara*

*University of Silesia, Katowice, Poland
**German Aerospace Center (DLR), Berlin

Logic Colloquium ASL2018, Udine, Italy

General relativity: Model of evolving Universe

- From Quantum State $\mathbb{L}(\mathcal{H})$ to Smooth spacetime 4-manifold $\mathbf{N}^4$

Let $\mathbf{N}^4 \simeq_{\text{homeom}} \mathbb{R}^4$. Then (under some extra conditions) it holds

If $\mathbb{L}(\mathcal{H})$ is nondistributive then $\mathbf{N}^4$ is exotic smooth R^4

- The model is realistic, i.e. it predicts the value of the cosmological constant from the curvature of R^4 as $\sim 10^{-29} g/cm^3$.
- Other models have problem with prediction of that value.

$(\mathbb{L}(\mathcal{H}), \vee, \wedge)$ the lattice of projections on closed linear subspaces of the Hilbert space $\mathcal{H}$
Exotic R^4 is nondiffeomorphic to the standard smooth $\mathbb{R}^4$

General relativity: Model of evolving Universe

- $\mathbb{L}(\mathcal{H})$ necessarily deals with random forcing:

If $\dim \mathcal{H} = \infty$ then

Maximal Boolean complete subalgebras B's in $\mathbb{L}(\mathcal{H})$ are (generically) atomless and are isomorphic with the measure algebra $B = \mathrm{Bor}(\mathbb{R})/\mathrm{Meager}$.

- V^B's – classical windows (local frames) to quantum world.
- V^B's – Boolean-valued models of ZFC each supporting random forcing.

- What is a meaning of this random forcing in cosmology?

Random forcing helps solving the problem of zero-modes – unacceptably big contributions to CC from QFT.

- Cohen forcing plays role in description of exotic R^4's appearing in the model.

Cohen forcing and exotic R^4 in the cosmological model – the Project

The cosmological model predicts:

- exotic R^4 at large scale
- QM at the smallest scales – $\mathbb{L}(\mathcal{H})$.

Based on this model, and on Cohen forcing, one aims at

$\mathrm{Aut}(P(\omega)/fin)$**-valued invariants of exotic R^4's are derivable from the lattice $\mathbb{L}$.**

- How to construct the invariant?

The outline of the construction

i) Include Cohen extension of the Goedel constructible universe $\mathbf{L}$, i.e. $\mathbf{L} \to \mathbf{L}[a]$, into a smooth countable atlas $\{U_i\}_{i \in I}$ of $\mathbb{R}^4$

$$\mathbf{L} \ni R^4_{\mathbf{L}} \simeq U_i \xrightarrow{[a]} U_j \simeq R^4_{\mathbf{L}[a]} \in \mathbf{L}[a]$$

ii) $\mathbf{L} \ni U_i \cap U_j \xrightarrow{[a]} U_i \cap U_j \in \mathbf{L}[a]$; $[a]$-arrow is the categorical morphism in $Sh(\{\mathbf{L}[a_i]\}_i)$ sheaves on the category of partially ordered Cohen extensions (inner models).

If a smoothness structure on $\mathbb{R}^4$ contains the morphism $[a]$, the smoothness has to be changed to another (exotic) $R^4_{[a]}$.

iii) Cohen real a is then represented as a trivial automorphism of $P(\omega)/fin$.

– a is determined by an (almost) permutation of ω performed on $0, 1, 0, 1, 0, 1, \ldots$ sequence, i.e. the trivial automorphism of $P(\omega)/fin$.

iv) $P(\omega)/fin$ is related with the lattice $\mathbb{L}$

– Calkin algebra $\mathcal{C} = \mathcal{B}(\mathcal{H})/\mathcal{K}$ extends $\mathcal{B}(\mathcal{H})$, bounded operators made by projections from $\mathbb{L}(\mathcal{H})$, and

$$P(\omega)/fin \hookrightarrow \mathcal{C}.$$

The automorphisms of certain factor III algebras derived from the lattice $\mathbb{L}$, e.g. $\mathrm{Aut}(\mathcal{C})$, distinguish exotic smooth R^4's .

Casson handles and $P(\omega)/fin$

- Casson handles (C.H.) are infinite geometric tree-like constructions in any small exotic R^4
- Any C.H. is embeddable in simple linear C.H.'s, e.g. $1, 0, 0, .., ..1, 1, 0, ...$

  Let this linear C.H. be a Cohen real a which is in $\mathrm{Aut}(P(\omega)/fin)$, then one conjectures

Small exotic R^4 gives rise to a diffeomorphic R^4 provided the original C.H.'s are replaced by C.H.'s still embeddable in the same linear C.H.'s as the original ones.

Casson handles and $P(\omega)/fin$

- Any infinite C.h. is extended to a hyperfinite $\widehat{\text{C.h.}}$ in nonstandard models of Peano arithmetic $M(\{\mathcal{U}_i\})_{i\in I}$ built by ultrafilters $\mathcal{U}_i$ on ω.
- C.h.'s of R^4 can be mapped one to the other by handle-slides without changing the smoothness of R^4, so that $\{\mathcal{U}_i\} \to \{\mathcal{U}_i\}$.

The relation with $\mathrm{Aut}(P(\omega)/fin)$ follows

$$(a : \{\mathcal{U}_i\} \to \{\mathcal{U}_i\}) \in \mathrm{Homeo}(\beta\omega \setminus \omega) \simeq \mathrm{Aut}(P(\omega)/fin)$$

where a – Cohen real.

Conclusions

- The cosmological model is a mathematical tool.
- Random and Cohen forcings are parts of the model
- One derives the invariant of exotic R^4's
- The cosmological model is a realistic physical model and solves some important problems in physics (CC).

- Król, J.; Asselmeyer-Maluga, T.; Bielas, K.; Klimasara, P. From Quantum to Cosmological Regime. The Role of Forcing and Exotic 4-Smoothness. *Universe* 2017, 3, 31.
- Asselmeyer-Maluga, T.; Król, J. How to obtain a cosmological constant from small exotic $\mathbb{R}^4$. *Phys. Dark Universe* 2018, 19, p. 66-77.
- Król, J., Model and set-theoretic aspects of exotic smoothness structures on $\mathbb{R}^4$, in At the Frontier of Spacetime, Fundamental Theories of Physics vol 183, pp. 217-240, Springer: Switzerland, 2016.
- Asselmeyer-Maluga, T., Bielas, K., Klimasara, P. and Król, J.: The latent meaning of forcing in quantum mechanics, Acta Phys. Pol. B Vol. 47, No 6 (2016) p. 1685-1690.

Thank You

for Your Attention