On categoricity of linear orders

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Definitions

Definition

A computable algebraic structure is called Δ^0_α-categorical if for any two computable copy of it there exists a Δ^0_α-isomorphism between of them.

Definition

A computable algebraic structure is called relatively Δ^0_α-categorical if for any two computable copy $\mathcal{L}_1$ and $\mathcal{L}_2$ of it there exists a $\Delta^x_\alpha(\mathcal{L}_1 \oplus \mathcal{L}_2)$-isomorphism between of them.

Computably and Δ^0_2-categorical and linear orders

Theorem[S.S. Goncharov and V.D. Dzgoev 1980; J.B. Remmel 1981]

A computable linear order is computably categorical iff it has finitely many successors.

Theorem [McCoy, 2003]

A computable linear order $\mathcal{L}$ with end points is relatively Δ^0_2-categorical iff $\mathcal{L}$ is a finite sum of finite linear orders, ζ, ω, ω^*, $1 + k \cdot \eta + 1$.

Theorem [McCoy, 2003]

If a linear order $\mathcal{L}$ has a computable copy with a computable successor relation, and computable left and right limit points then $\mathcal{L}$ is Δ^0_2 categorical iff it is relatively Δ^0_2 categorical.

Δ^0_n categorical linear orders

Theorem[C. Ash, 1986]

If an ordinal α such that $\omega^{\delta+n} \leq \alpha < \omega^{\delta+n+1}$ then α is $\Delta^0_{\delta+2n}$ categorical, and is not Δ^0_β categorical for any $\beta < \delta + 2n$.

Theorem[N. Bazenov, 2016]

If an ordinal α such that $\omega^n \leq \alpha < \omega^{n+1}$ then $\emptyset^{2n-1}$ is the degree of categoricity of α.
If an ordinal α such that $\omega^{\delta+n} \leq \alpha < \omega^{\delta+n+1}$ $(\delta \geq \omega)$ then $\emptyset^{\delta+2n}$ is the degree of categoricity of α.

Scattered linear orders

Definition

1. $\mathbf{VD_0} = \{\mathbf{0}, \mathbf{1}\}$;
2. $\mathbf{VD}_\alpha = \{\sum_{\mathbf{i} \in \tau} \mathcal{L}_\mathbf{i} \mid \mathcal{L}_\mathbf{i} \in \bigcup_{\beta<\alpha} \mathbf{VD}_\beta,\ \tau \in \{\omega, \omega^*, \zeta, \mathbf{0}, \mathbf{1}, \mathbf{2}, \ldots\}\}$.

Тогда $VD = \bigcup_\alpha \mathbf{VD}_\alpha$.

The least ordinal α such that $\mathcal{L} \in VD_\alpha$ is called the VD-rank of $\mathcal{L}$.

Definition

The least ordinal α such that $\mathcal{L}$ is a finite sum of linear orders with VD-rank less or equal than α is called the VD^*-rank of $\mathcal{L}$.

The VD-rank of a scattered linear order $\mathcal{L}$ is equal to the Hausdorff rank of $\mathcal{L}$.

Upper bounds

Theorem [A. Frolov, Z.]

If a scattered linear order $\mathcal{L}$ has VD^*-rank $\delta + n$, where δ is a constructive limit ordinal, n is finite ordinal, then $\mathcal{L}$ is relatively $\Delta^0_{\delta+2n}$ categorical.

Examples with sharp bounds

Theorem [A. Frolov, Z.]

For any constructive ordinal $\delta + n \geq 2$, where δ is a constructive limit ordinal, n is finite ordinal and any β such that $3 \leq \beta \leq \delta + 2n$, and β is not a successor of a limit ordinal there exist a computable scattered linear order $\mathcal{L}$ with rank $\delta + n$ which is relatively Δ^0_β categorical and is not Δ^0_γ categorical for any $\gamma < \beta$.

Δ^0_2-categoricity

Question

Are Δ^0_2-categorical linear orders and relatively Δ^0_2-categorical linear orders the same classes?

Conjecture

If linear order $\mathcal{L}$ has a computable copy with $\emptyset'$-computable block relation then $\mathcal{L}$ Δ^0_2-categorical iff relatively Δ^0_2-categorical.

Conjecture

If linear order $\mathcal{L}$ has a computable copy with $\emptyset'$-computable the left limit point and the right limit point relations then $\mathcal{L}$ Δ^0_2-categorical iff relatively Δ^0_2-categorical.

Δ_2^0-categoricity

Theorem [M. Eryashkin, Z.]

A computable strongly η-like linear order is relatively Δ_2^0-categorical iff it is Δ_2^0-categorical.

Theorem [M. Eryashkin, Z.]

A computable discreet linear order is relatively Δ_2^0-categorical iff it is Δ_2^0-categorical.