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Tree-shaped one-pass tableau systems for Linear Temporal Logic satisfiability checking

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Joint work with Angelo Montanari, Mark Reynolds, Luca Geatti

The need for formal verification

Safety-critical systems need to avoid bugs at all costs.

Formal Verification develops automatic techniques to provide mathematical proofs of software correctness.

Formal verification and Logic

In formal verification, an abstract model of the system is checked against a formal specification of the desired behavior.

- Systems are usually modeled as automata.
- Specifications are usually expressed as temporal logic formulas.

This is the model checking problem.

Linear Temporal Logic

Linear Temporal Logic (LTL) is a propositional modal logic commonly used as specification language.

$\mathsf{X}\,\alpha$	α will be true at the next state.
$\alpha\,\mathcal{U}\,\beta$	β will eventually be true, and α always holds until then.
$\mathsf{F}\,\beta \equiv \top\,\mathcal{U}\,\beta$	β will eventually be true.
$\mathsf{G}\,\beta \equiv \neg\,\mathsf{F}\neg\beta$	β will always be true.

Linear Temporal Logic (2)

If infinitely many requests are received,
then infinitely many replies are sent.

$$\mathrm{G}\,\mathrm{F}\,r \longrightarrow \mathrm{G}\,\mathrm{F}\,q$$

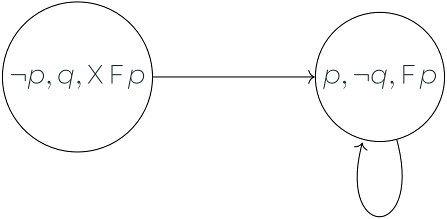
LTL satisfiability checking

LTL satisfiability is the problem of checking whether there exists a model that satisfies a given LTL formula.

- Important to check consistency of specifications.
- Seemingly very hard problem: PSPACE-complete.
- Many algorithmic solutions to solve it:
  - (Büchi) Automata-based
  - Tableau methods
  - Temporal resolution
  - Reduction to model checking
  - ...

Tableaux-based methods for LTL satisfiability

Common tableaux-based methods for LTL are graph-shaped and require multiple passes to be built and checked for successful paths.



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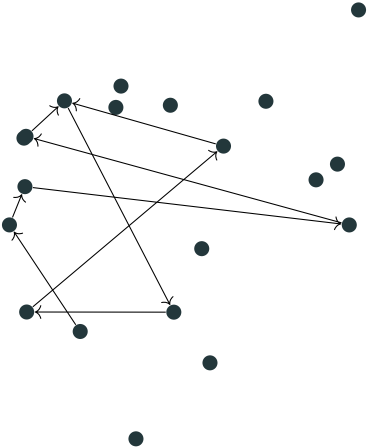

A One-Pass Tree-Shaped Tableau for LTL

A one-pass tree-shaped tableau method for LTL satisfiability was recently proposed.

GandALF 2016

M. Reynolds. "A New Rule for LTL Tableaux." In: *Proc. of the 7th International Symposium on Games, Automata, Logics and Formal Verification*. GandALF 2016

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A one-pass tree-shaped tableau method for LTL satisfiability was recently proposed, implemented.

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M. Bertello, N. Gigante, A. Montanari, and M. Reynolds. "Leviathan: A New LTL Satisfiability Checking Tool Based on a One-Pass Tree-Shaped Tableau." In: *Proc. of the 25th International Joint Conference on Artificial Intelligence*. IJCAI 2016

`http://www.github.com/corralx/leviathan`

A One-Pass Tree-Shaped Tableau for LTL

A one-pass tree-shaped tableau method for LTL satisfiability was recently proposed, implemented, and parallelized.

GandALF 2017

John Christopher McCabe-Dansted and Mark Reynolds. "A Parallel Linear Temporal Logic Tableau." In: *Proceedings 8th International Symposium on Games, Automata, Logics and Formal Verification.* 2017, pp. 166–179

A One-Pass Tree-Shaped Tableau for LTL

A one-pass tree-shaped tableau method for LTL satisfiability was recently proposed

- Purely tree-shaped rule-based search procedure.
- A single pass is sufficient to determine the acceptance of rejection of a given branch.
- Very simple set of rules, combining the simplicity of declarative tableaux with the efficiency of one-pass systems.
- Easy to extend.
- Easy to parallelize with huge speedups.

Example

$\{\mathsf{G}\,\mathsf{F}(p \wedge \mathsf{X}\,\neg p)\}$

Example

$$\{\mathrm{G}\,\mathrm{F}(p \wedge \mathrm{X}\,\neg p)\}$$

$$\{\,\mathrm{F}(p \wedge \mathrm{X}\,\neg p),\, \mathrm{X}\,\mathrm{G}\,\mathrm{F}(p \wedge \mathrm{X}\,\neg p)\}$$

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$\ldots$

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$\{\neg p, \mathsf{G}\,\mathsf{F}(p \wedge \mathsf{X}\,\neg p)\}$

Example

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$$\{\neg p, p,\ \mathsf{X}\,\neg p, \ldots\}$$

✗

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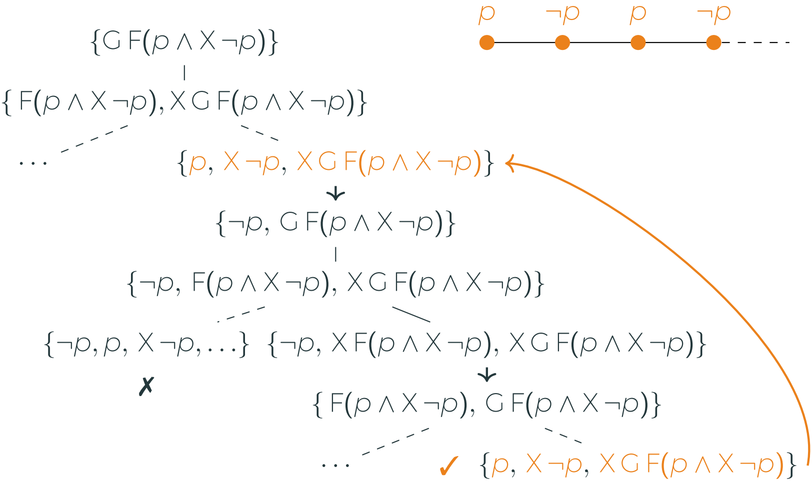
Example


p ¬p p ¬p
{G F(p ∧ X ¬p)}
{ F(p ∧ X ¬p), X G F(p ∧ X ¬p)}
. . .
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{¬p, p, X ¬p, . . .}
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✗
{ F(p ∧ X ¬p), G F(p ∧ X ¬p)}
. . .
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Example - unsatisfiable formula

$$\{\mathsf{G}\,\neg p \land q\,\mathcal{U}\,p\}$$

Example - unsatisfiable formula

$$\{\mathsf{G}\,\neg p \wedge q\,\mathcal{U}\,p\}$$

$$|$$

$$\{\,\mathsf{G}\,\neg p, q\,\mathcal{U}\,p\}$$

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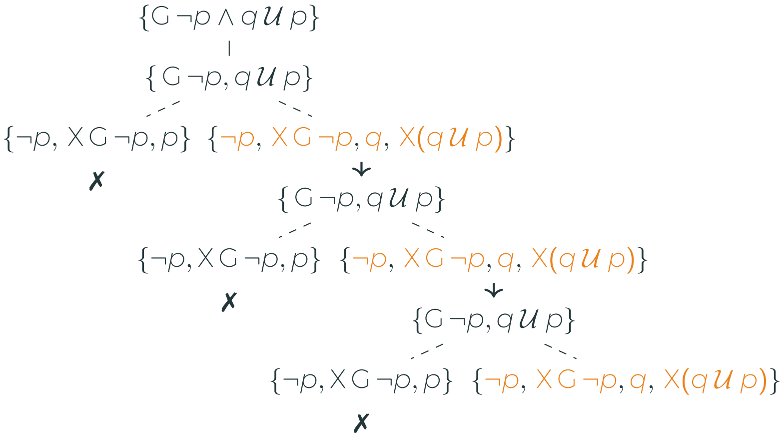

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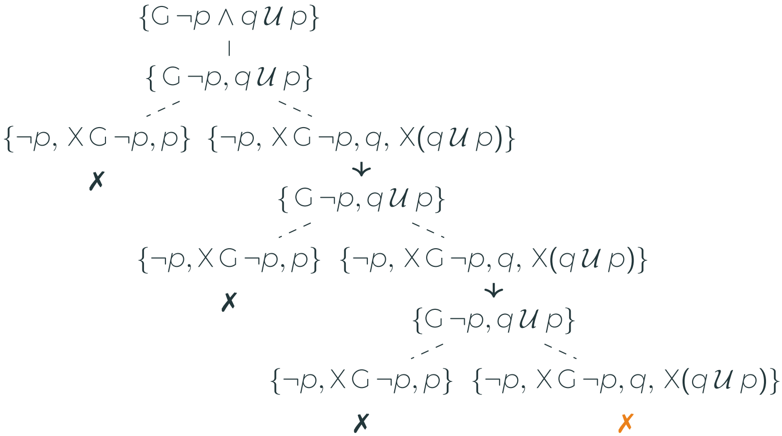

Beyond LTL

Our conjecture is that the simplicity of the tableau can be adapted to more expressive logics.

Extensions developed so far:

- LTL with past operators;

LPAR 21

Nicola Gigante, Angelo Montanari, and Mark Reynolds. "A One-Pass Tree-Shaped Tableau for LTL+Past." In: *21st Int. Conference on Logic for Programming, Artificial Intelligence and Reasoning*. 2017, pp. 456–473

Beyond LTL

Our conjecture is that the simplicity of the tableau can be adapted to more expressive logics.

Extensions developed so far:

- LTL with past operators;
- Timed Propositional Temporal Logic (TPTL) and Bounded TPTL + Past ($TPTL_b$+P)

Submitted

Luca Geatti, Nicola Gigante, Angelo Montanari, and Mark Reynolds. "One-pass and tree-shaped tableau systems for TPTL and $TPTL_b$+P." In: *Submitted for review.* 2018

Beyond LTL

Our conjecture is that the simplicity of the tableau can be adapted to more expressive logics.

Future and in progress work:

- extension to full TPTL with past;
- translation of tableau rules into new SAT/SMT encodings for the LTL satisfiability problem;
- study of the model-theoretic properties of models found by the tableau PRUNE rule.

Future work

Future lines of work:

- Add the past to our satisfiability checking tool.
    - Not trivial: our rule causes a lot of backtracking
- Exploit the modular structure of the tableau to extend it to other LTL extensions.
- Implement these extensions: one tool for a broad family of linear time logics.

Thank you!

Questions?

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[Ber+16] M. Bertello, N. Gigante, A. Montanari, and M. Reynolds. "Leviathan: A New LTL Satisfiability Checking Tool Based on a One-Pass Tree-Shaped Tableau." In: *Proc. of the 25th International Joint Conference on Artificial Intelligence*. IJCAI 2016.

[Gea+18] Luca Geatti, Nicola Gigante, Angelo Montanari, and Mark Reynolds. "One-pass and tree-shaped tableau systems for TPTL and $TPTL_b$+P." In: *Submitted for review*. 2018.

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[MR17] John Christopher McCabe-Dansted and Mark Reynolds. "A Parallel Linear Temporal Logic Tableau." In: *Proceedings 8th International Symposium on Games, Automata, Logics and Formal Verification*. 2017, pp. 166–179.

[Rey16] M. Reynolds. "A New Rule for LTL Tableaux." In: *Proc. of the 7th International Symposium on Games, Automata, Logics and Formal Verification*. GandALF 2016.

Why three occurrences?

Consider this formula:

$$\phi \equiv p \land \mathsf{G}(p \longleftrightarrow \mathsf{X}\,\neg p) \land \mathsf{G}\,\mathsf{F}\,q_1 \land \mathsf{G}\,\mathsf{F}\,q_2 \land \mathsf{G}\,\neg(q_1 \land q_2) \land \mathsf{G}(q_1 \longrightarrow \neg p) \land \mathsf{G}(q_2 \longrightarrow \neg p)$$

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$$\phi \equiv p \wedge \mathrm{G}(p \longleftrightarrow \mathrm{X}\,\neg p) \wedge \mathrm{G}\,\mathrm{F}\,q_1 \wedge \mathrm{G}\,\mathrm{F}\,q_2 \wedge$$
$$\mathrm{G}\,\neg(q_1 \wedge q_2) \wedge \mathrm{G}(q_1 \longrightarrow \neg p) \wedge \mathrm{G}(q_2 \longrightarrow \neg p)$$

And its tableau:

$$\{\phi\}$$

$$\{\ldots, p, \mathrm{X}\,\mathrm{F}\,q_1, \mathrm{X}\,\mathrm{F}\,q_2, \ldots\}$$

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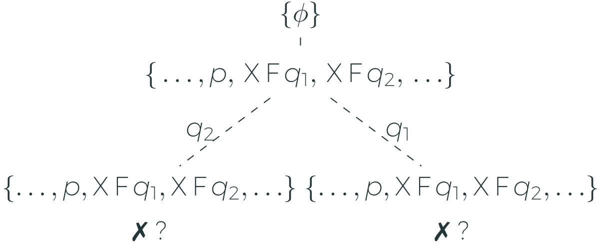

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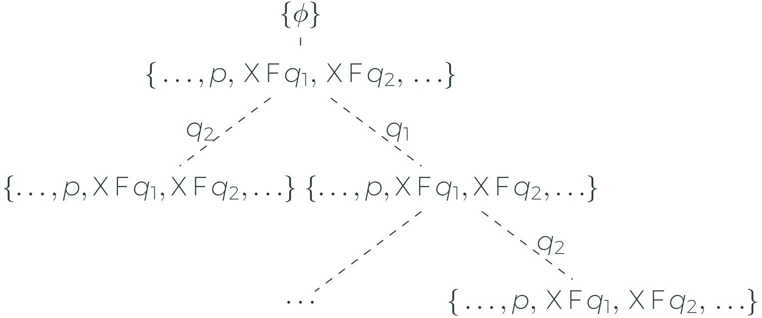

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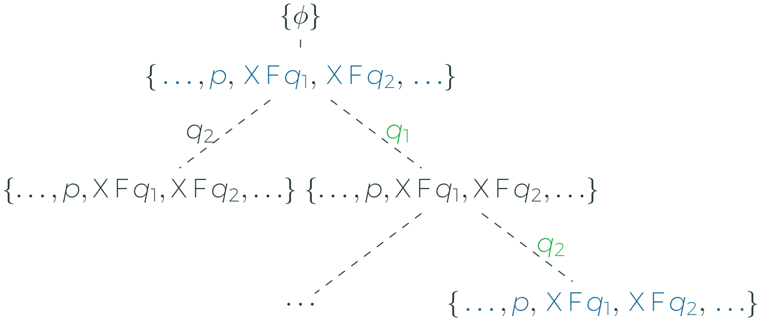

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