Description logics, ontologies, and automated reasoning: an introduction

— Day 1, Part 1 —

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Welcome!

Thanks for having me!

I hope we have good time and learn a lot.

Thanks to **Thomas Schneider**: I am recycling some of our slides from an ESSLLI course.

Welcome!

Let me know if you

... have questions. – **Do ask** them at any time.

... have difficulties understanding me or reading my writing/...

This is course a bit **interactive**: you may have to

... think

... answer questions

... do mini exercises

What's in this course?

1. Introduction
    - The basic DL $\mathcal{ALC}$, reasoning problems
    - Relation with other logics, ontologies, examples and exercises
2. DLs, ontologies, and OWL: applications and tools
3. Core reasoning tasks & selected results on
    - undecidability & lower bounds
    - upper bounds & a bit of model theory
4. Other reasoning tasks:
    - explaining entailments
    - justifications and more

Plan for today

1. DL basics
2. Relationship with other logics
3. Ontologies
4. OWL and DLs

DLs: the core

Core part of a DL: its **concept language**, e.g.:

$$\texttt{Animal} \sqcap \exists\, \texttt{hasPart.Feather}$$

describes all animals that are related via "hasPart" to a feather.

Syntactic ingredients of a concept language:

- **Concept names** stand for **sets of elements**, e.g., Animal
- **Role names** stand for **binary relations**, e.g., hasPart
- **Constructors** to build **concept expressions**, e.g., $\sqcap$, $\exists$

Syntax and semantics of $\mathcal{ALC}$

Semantics given by means of an **interpretation** $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, where

- $\Delta^{\mathcal{I}}$ is a nonempty set (the **domain**), and
- $\cdot^{\mathcal{I}}$ is a mapping (the **interpretation function**) as follows:

Constructor	Syntax	Example	Semantics
concept name	A	`Human`	$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
role name	r	`likes`	$r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$

For C, D concepts and R a role name:

conjunction	$C \sqcap D$	`Human` $\sqcap$ `Male`	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$
disjunction	$C \sqcup D$	`Nice` $\sqcup$ `Rich`	$C^{\mathcal{I}} \cup D^{\mathcal{I}}$
negation	$\neg C$	$\neg$`Meat`	$\Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
restrictions:			
existential	$\exists r.C$	$\exists$`hasChild.Human`	$\{x \mid \exists y.(x,y) \in r^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\}$
value	$\forall r.C$	$\forall$`hasChild.Blond`	$\{x \mid \forall y.(x,y) \in r^{\mathcal{I}} \Rightarrow y \in C^{\mathcal{I}}\}$

Understanding syntax and semantics of $\mathcal{ALC}$

We can "draw" interpretations . . .
(similarly to Kripke models if you happen to know modal logic)

Exercise 1: Using concepts `Person`, `Happy`, `Pet`, `Cat`, `Dog` and a role name `owns`, formulate $\mathcal{ALC}$ concepts that describe

1. happy pet owners
2. unhappy pet owners who own an old cat
3. pet owners who own a cat, a dog, and only cats and dogs

For these concepts,
draw an interpretation with an instance of that concept.

Basic reasoning problems in $\mathcal{ALC}$

Definition: let C, D be $\mathcal{ALC}$ concepts. We say that

- $e \in C^\mathcal{I}$ is **an instance of** C in $\mathcal{I}$.
- C is **satisfiable** if there is an interpretation $\mathcal{I}$ with $C^\mathcal{I} \neq \emptyset$.
- C is **subsumed by** D (written $\emptyset \models C \sqsubseteq D$) if, for every interpretation $\mathcal{I}$, we have that $C^\mathcal{I} \subseteq D^\mathcal{I}$.

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Exercise 2: Which of the following concepts is satisfiable? Which is subsumed by which?

(1) $\exists r.(A \sqcup \neg A)$ (2) $\exists r.A \sqcap \forall r.\neg A$
(3) $\exists r.A \sqcap \forall s.\neg A$ (4) $\forall r.(A \sqcap \neg A)$

The TBox

The "class-level", terminological part of our knowledge base:

Definition

- A **general concept inclusion (GCI)** has the form $C \sqsubseteq D$, for C, D (possibly complex) concepts
- A **general TBox** is a finite set of GCIs: $\mathcal{T} = \{C_i \sqsubseteq D_i \mid 1 \leqslant i \leqslant n\}$
- $\mathcal{I}$ **satisfies** $C \sqsubseteq D$ if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ (written $\mathcal{I} \models C \sqsubseteq D$)
- $\mathcal{I}$ is a **model of TBox** $\mathcal{T}$ if $\mathcal{I}$ satisfies every $C_i \sqsubseteq D_i \in \mathcal{T}$
- We use $C \equiv D$ to abbreviate $C \sqsubseteq D$, $D \sqsubseteq C$

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Example:

$$\begin{aligned} \{\ \texttt{Father} &\equiv \texttt{Man} \sqcap \exists\,\texttt{hasChild.Human}\,, \\ \texttt{Human} &\equiv \texttt{Mammal} \sqcap \forall\,\texttt{hasParent.Human}\,, \\ \exists\,\texttt{favourite.Brewery} &\sqsubseteq \exists\,\texttt{drinks.Beer}\ \} \end{aligned}$$

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Exercise 3: Draw a model of the above TBox.
Draw an interpretation that is **not** a model of it.

Reasoning problems with respect to a TBox

Definition: let C, D be concepts, $\mathcal{T}$ a TBox. We say that

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Example:

$$\mathcal{T} = \{ \begin{array}{rcl} A & \sqsubseteq & B \sqcap \exists r.C\,, \\ \exists r.\top & \sqsubseteq & \neg A \end{array} \}$$

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Exercise 4: Does $\mathcal{T}$ have a model?
Are all concept names in $\mathcal{T}$ satisfiable?
Any subsumptions that you can point out?

How many models does a TBox have?

The ABox

TBox
- captures knowledge on a general, conceptual level
- contains concept def.s + general axioms about concepts

ABox
- captures knowledge on an individual level
- is a finite set of
  - **concept assertions** $a : C$ e.g., $\texttt{John} : \texttt{Man}$, and
  - **role assertions** $(a, b) : r$ e.g., $(\texttt{John}, \texttt{Mary}) : \texttt{hasChild}$

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Semantics: an interpretation $\mathcal{I}$

- maps each **individual name** e to some $e^{\mathcal{I}} \in \Delta^{\mathcal{I}}$
- satisfies a concept assertion $a : C$ if $a^{\mathcal{I}} \in C^{\mathcal{I}}$
- satisfies a role assertion $(a, b) : r$ if $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$
- is a **model** of an ABox $\mathcal{A}$ if $\mathcal{I}$ satisfies each assertion in $\mathcal{A}$
- $a : C$ is **entailed by** $\mathcal{A}$ if every model of $\mathcal{A}$ satisfies $a : C$

The ABox

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repeated from previous slide

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Example:

$$\mathcal{A} = \{ \quad a : (B \sqcap \exists r.C), \quad b : (A \sqcap \neg P \sqcap \forall s.\forall r.F), \quad (b, a) : s \}$$

The ABox

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Exercise 5: Does $\mathcal{A}$ have a model? – Describe some of them.
Can you see any entailments?

(Later) Can you translate this into FOL? ML?

Ontologies: TBox and ABox

Definition: an **ontology** consists of

- a TBox that captures knowledge on a general, conceptual level
- an ABox that captures knowledge on an individual level and **uses terms described in the TBox**

Notation: $(\mathcal{T}, \mathcal{A})$ or $\mathcal{T} \cup \mathcal{A}$ – no difference!

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Notation: $(\mathcal{T}, \mathcal{A})$ or $\mathcal{T} \cup \mathcal{A}$ – no difference!

Semantics:

- $\mathcal{I}$ is a **model** of $\mathcal{O} = (\mathcal{T}, \mathcal{A})$ (written $\mathcal{I} \models \mathcal{O}$)
  if $\mathcal{I}$ satisfies each assertion and axiom in $\mathcal{O}$
  alternatively: $\mathcal{I} \models \mathcal{T}$ and $\mathcal{I} \models \mathcal{A}$
- $\mathcal{O}$ is **consistent** if it has a model
- $\mathcal{O}$ is **coherent** if each conc. name A in $\mathcal{O}$ is satisfiable w.r.t. $\mathcal{O}$
- $C \sqsubseteq D$ is **entailed by** $\mathcal{O}$ if every model of $\mathcal{O}$ satisfies $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
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Ontologies: TBox and ABox

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Exercise 6: Does $\mathcal{O}$ have a model? – Describe some of them.
Can you see any entailments?
What about $\mathcal{O} \cup \{b : C\}$ or $\mathcal{O} \cup \{b : A\}$?

Ontologies: TBox and ABox

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repeated from previous slide

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Lemma

$C \sqsubseteq D$ is entailed by $\mathcal{O} = (\mathcal{T}, \mathcal{A})$ iff $C \sqsubseteq D$ is entailed by $\mathcal{T}$.

Ontologies: TBox and ABox

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repeated from previous slide

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Lemma

$C \sqsubseteq D$ is entailed by $\mathcal{O} = (\mathcal{T}, \mathcal{A})$ iff $C \sqsubseteq D$ is entailed by $\mathcal{T}$.

Proof: for "$\Leftarrow$", note that every model of $\mathcal{O}$ is one of $\mathcal{T}$.

For "$\Rightarrow$", use contraposition; distinguish between $\mathcal{O}$ being inconsistent (trivial) and consistent (combine a model witnessing $\mathcal{T} \not\models C \sqsubseteq D$ and one of $\mathcal{O}$ to one witnessing $\mathcal{O} \not\models C \sqsubseteq D$). $\square$

Relationship between Reasoning Problems

Theorem

Let C, D be possibly complex concepts, $\mathcal{O} = (\mathcal{T}, \mathcal{A})$ an ontology, and b an individual name.

- C is satisfiable w.r.t. $\mathcal{T}$ iff $(\mathcal{T}, \{b : C\})$ is consistent.
- $\mathcal{T} \models C \sqsubseteq D$ iff $(\mathcal{T}, \{b : C \sqcap \neg D\})$ is **not** consistent.
- $\mathcal{O} \models b : C$ iff $(\mathcal{T}, \mathcal{A} \cup \{b : \neg C\})$ is **not** consistent.

. . .

Let's switch to next slide set for **Ontologies and other logics**. . .